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Hierarchical neutrino mass matrices, CP violation and leptogenesis

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Abstract: In this work we study examples of hierarchical neutrino mass matrices inspired by family symmetries, compatible with experiments on neutrino oscillations, and for which there is a connection among the low energy CP violation phase associated to neutrino oscillations, the phases appearing in the amplitude of neutrinoless double beta decay, and the phases relevant for leptogenesis. In particular, we determine the predictions from a texture based on an underlying SU(3) family symmetry together with a GUT symmetry, and a strong hierarchy for the masses of the heavy right handed Majorana masses. We also give some examples of inverted hierarchies of neutrino masses, which may be motivated in the context of U(1) family symmetries.

Keywords: Beyond Standard Model, Neutrino Physics, CP violation.

Contents

1. Introduction

This work is motivated by the study, in the context of models accommodating the masses of leptons, of the correlation among the parameters associated to CP violation, appearing in neutrino oscillations, CP violation in leptogenesis and CP violation appearing in neutrinoless double beta decay ($\beta\beta_{\nu_0}$). In particular we study the kind of models presented in [1] (which are based on an underlying SU(3) family symmetry together with a GUT symmetry and a strong hierarchy for the masses of the heavy right handed Majorana masses) and

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also models giving inverted hierarchies for neutrino masses, which can be understood in the context of a U(1) family symmetry.

Recently there have appeared some studies [2]–[5] of a possible connection between low energy phases and those phases relevant for leptogenesis, motivated by the fact that leptogenesis is a very attractive candidate in explaining the baryon asymmetry of the universe (BAU) and also by the present information coming from neutrino oscillation experiments. In the leptogenesis scenario a $B-L$ asymmetry is produced from the decay of the heavy right-handed Majorana neutrinos, N_j. This asymmetry is parameterized in terms of asymmetry parameters, ε_j, which can be expressed in terms of the heavy right handed Majorana masses and the Yukawa couplings for neutrinos. Therefore, in the context of models describing the correct neutrino mass splittings and mixings, it is natural to look for such a connection and for a correlation between the sign of the baryon number of the universe and the strength of the CP violation in neutrino oscillations. Such correlations are not a general feature of the models explaining neutrino oscillations and in some cases are quite model dependent. Nevertheless, if there are plausible models for family symmetries explaining not only neutrino oscillations but also the low energy parameters of quarks, it is interesting to examine whether or not these models are compatible with the leptogenesis scenario and whether or not it is possible to establish correlations among CP asymmetries appearing in leptogenesis, the parameters of CP violation in neutrino oscillations and the phases appearing in the neutrinoless double beta decay. The relevance of neutrinoless double decay processes is that not only the Majorana nature of neutrinos can be unveiled, since their amplitude is proportional to an average mass containing the Majorana phases of neutrinos, but also that the scale of neutrino masses may be determined.

Neutrino oscillation experiments have provided evidence for non-vanishing neutrino masses and mixings. Two years ago there were yet considered many possibilities to explain the solar neutrino mixing, now the recent results from KamLAND [6] indicate that the most favoured solution is the MSW LMA solution. This information and that coming from the atmospheric mixing analyses [7]–[10], which may be summarized as follows

$$\begin{aligned}
\tan^2\theta_{\rm atm} &= 1^{-0.57}_{+1.3}\,, \qquad \tan^2\theta_{\rm sol} = (4^{-1.7}_{+3.9})\times 10^{-1}\,,\\
\tan^2\theta_{\rm rct} &< 5.5\times 10^{-2}\,,\\
\Delta m^2_{\rm sol} &\in (5.1, 9.7)\times 10^{-5}\,{\rm eV}^2\,, \qquad \Delta m^2_{\rm atm} \in (1.2, 4.8)\times 10^{-3}\,{\rm eV}^2\,,
\end{aligned} \tag{1.1}$$

has given us a definite point of departure from the Standard Model (SM)[1] and thus a way to probe possible symmetries for leptons and quarks in an unified scheme. With this information it has been possible to carry out *bottom-up* approach analyses [11, 12, 13] in order to reconstruct the possible forms of the effective neutrino mass matrix. The most plausible forms consistent with data are: (i) *hierarchical (canonical)*, which can be such that $m_1 \ll m_2 \ll m_3$ or $m_1 \lesssim m_2 \ll m_3$, (ii) *inverted hierarchical* $m_2 \gtrsim m_1 \gg m_3$ or (iii) *degenerate* $m_1^2 \approx m_2^2 \approx m_3^2$.

[1]Here $\theta_{\rm atm}$, $\theta_{\rm sol}$, $\theta_{\rm rct}$ are the mixing angles describing atmospheric, solar, and reactor neutrino oscillation experiments, respectively.

A direct way to re-construct the possible forms of the neutrino mass matrix is by working in the *flavour basis* in which the charged lepton mass matrix is diagonal and the mixings appearing in the lepton mixing matrix, $U_{\rm MNS}$ matrix are only due to neutrinos. However, in trying to identify the possible broken symmetries underlying the neutrino 'puzzle', it is convenient to extract the neutrino mass matrix in the *symmetry basis*, in which the patterns of the possible (broken) symmetries underlying the leptons is reflected on the structure of the mass matrices. In this basis, charged lepton and neutrino mass matrices may not be diagonal and hence we can study the possible contributions of charged leptons and neutrinos to the $U_{\rm MNS}$ matrix, given by different family symmetries.

The work is organized as follows. In section 2 we review the basic structure of a family symmetry motivated by an $\mathrm{SU}(3)_F \times \mathrm{SO}(10)_{\rm GUT}$ symmetry [1] and the general structure of $\mathrm{U}(1)_F$ family symmetries. In section 3, we comment upon the diagonalization of hierarchical mass matrices, which can be used for canonical and inverted hierarchies, discussing the details in appendix A. In this section we also construct the $U_{\rm MNS}$ in terms of those matrices diagonalizing neutrino and charged lepton mass matrices. In section 4 we derive the predictions for the CP violation phase appearing in neutrino oscillations, $\delta_{\rm O}$, for two different kinds of hierarchies of the neutrino mass matrix (one producing the hierarchy $m_{\nu_3} \gg m_{\nu_2} \gg m_{\nu_1}$ and the other producing the hierarchy $m_{\nu_2} \gtrsim m_{\nu_1} \gg m_{\nu_3}$). We give an explicit realization of each one, motivated by the $\mathrm{SU}(3)_F \times \mathrm{SO}(10)_{\rm GUT}$ and the $\mathrm{U}(1)_F$ family symmetries, respectively. In section 5 we determine that these hierarchies are compatible with the leptogenesis scenario, giving an approximate value of the baryon asymmetry produced, and comment upon the connections between the phases appearing in neutrino oscillations and the phases relevant for leptogenesis. In section 6 we determine the Majorana phases for the hierarchies presented and comment further upon the relations to the leptogenesis phase. We conclude with a summary and outlook.

2. Family symmetries and symmetry basis

The possibility of explaining the masses of quarks and leptons through a set of symmetries containing the SM model has been widely explored. Such explanation may be achieved within the context of more fundamental theories such as String Theory, Grand Unified theories and Flavour symmetries (those symmetries distinguishing between families) also called *horizontal symmetries*. We consider models in which there is an underlying family symmetry and the neutrino masses are given by the see-saw mechanism [14]–[17]. When these symmetries are broken they leave an imprint in the form of the mass matrices appearing in the effective mass lagrangian. In the leptonic sector, this has the form

$$-L_m^\ell = \bar{\nu^o}_L m_D^\nu \nu_R^o + \frac{1}{2}\bar{\nu^c}_R M_R \nu_R + \bar{l^o}_L m^l l_R^o + \text{h.c.} \tag{2.1}$$

$$= \frac{1}{2}\bar{n^c}_L m_{LL}^\nu n_L + \bar{l^o}_L m^l l_R^o + h.c., \tag{2.2}$$

where ν_R^o labels the right-handed (R-H) Majorana neutrinos, l^o the charged leptons and we have assumed that the possible Majorana mass term associated with the left-handed

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neutrinos, ν_L^o, vanishes. The state $n_L = (\nu_L^o, (\nu^o)_R^c)$, has an effective mass m_{LL}^ν given, approximately, by the see-saw formula [14]–[17],

$$m_{LL}^{\nu\dagger} \approx -m_D^\nu M_R^{-1} (m_D^\nu)^t \,. \tag{2.3}$$

Each matrix in eq. (2.1) is not necessarily diagonal, showing the patterns left by the broken family symmetries. This basis defines what we call the *symmetry basis.*

The most plausible patterns [11, 12, 13] describing the effective neutrino mass matrix m_{LL}^ν are the given by the following patterns, namely the hierarchical pattern

$$\mathrm{H}: \quad m_{LL}^\nu = \begin{pmatrix} \epsilon' & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \frac{\tilde{m}_\nu}{2} \,, \tag{2.4}$$

which produces the ordering $m_{\nu_3} \gg m_{\nu_2} > m_{\nu_1}$, the inverted hierarchical patterns:

$$\begin{aligned} \mathrm{I}H1: \quad m_{LL}^\nu &= \begin{pmatrix} \epsilon' & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \epsilon & \epsilon \\ \frac{1}{\sqrt{2}} & \epsilon & \epsilon \end{pmatrix} \tilde{m}_\nu \,, \qquad \mathrm{IH2}: \quad m_{LL}^\nu = \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & \frac{1}{2} & \frac{1}{2} \\ \epsilon & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \tilde{m}_\nu \,, \\ \tilde{m}_\nu &= \mathcal{O}(\max\{m_{\nu_3}, m_{\nu_2}, m_{\nu_1}\}) \,, \end{aligned} \tag{2.5}$$

which gives rise to the ordering $m_{\nu_3} \ll m_{\nu_1} \lesssim m_{\nu_2}$, and, finally, the anarchical patterns, in which none of the elements are related and may give rise to different spectra. Here $\epsilon, \epsilon' \ll 1$ and we have indicated only the order of magnitude without specifying the order 1 coefficients. The hierarchies H and $IH2$ can be realized in the flavour basis, where the charged leptons are diagonal, reproducing the observed lepton mixings and neutrino mass splittings. The inverted hierarchy $IH2$ typically gives a negligible mixing angle, θ_{13}^ν [18]. The hierarchy $IH1$ produces a rather small angle θ_{12}^ν, unless there is a fine tunning in the parameters. The three hierarchies may be realized in suitable models accommodating neutrinos and leptons, where the mixing angles arising from the diagonalization of m_{LL}^ν receive contributions coming from the diagonalization of m^l and especially those from θ_{rct}, as we shall shortly see in section 4.1.

Although there have been several proposals of symmetries capable of accommodating these different possibilities [19, 20, 21, 18, 22, 23], only some of them [20, 22, 1] have been given in the context of a more general model for quarks and leptons. We are interested in knowing whether in models of this kind, there are correlations among the phases appearing in neutrino oscillations, neutrinoless double beta decay and leptogenesis. To be more concrete, we would like to analyze predictions for:

(I) A class of hierarchies of the type $H1$ and detailed predictions for an specific example that may be realized in the context of an $\mathrm{SU}(3)_F \times \mathrm{SO}(10)_{GUT}$ symmetry,

(II) a hierarchy of the type $IH2$ in the context of $\mathrm{U}(1)_F$ symmetries.

Before we present the predictions for the CP violation phases, and in order to motivate the hierarchies H, $IH1$ and $IH2$ in the context of family symmetries, we review the general features of the family symmetries $\mathrm{SU}(3)_F$ and U_F that we consider.

The case of a texture inspired in a $\mathrm{SU}(3)_F \times \mathrm{SO}(10)$ symmetry. In [1] we have considered a texture for quarks and leptons inspired by a supersymmetric $\mathrm{SU}(3)_F \times \mathrm{SO}(10)_{\mathrm{GUT}}$[2] family symmetry where we have parameterized the Yukawa matrices for fermions by

$$Y^f = \frac{m_D^f}{(m_D^f)_{(3,3)}} = \begin{pmatrix} \varepsilon^8 & \varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^3(z+(x-y)\varepsilon) \\ -\varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^2(a_f w + u\varepsilon) & \varepsilon^2(a_f w - u\varepsilon) \\ -\varepsilon^3(z+(x-y)\varepsilon) & \varepsilon^2(a_f w - u\varepsilon) & 1 \end{pmatrix}, \tag{2.6}$$

here the index f denotes the kind of fermions: $f = \nu, u, d, l$. In the context of a supersymmetric theory of fermion masses we may construct the Yukawa superpotential based on the terms allowed by the symmetry; for example for eq. (2.6), the dominant terms are given by the operators [34]

$$\left(\frac{1}{M_3^2}\psi_i \phi_3^i \psi_j^c \phi_3^j + \frac{1}{M^2}\psi_i \phi_{23}^i \psi_j^c \phi_{23}^j\right) H_\alpha , \quad i,j = 1,2,3 \tag{2.7}$$

where ψ_i and ψ_i^c are left-handed quarks and leptons ($\psi_i \in (Q_i, L_i)$, $\psi_i^c \in (U_i^c, D_i^c, E_i^c, N_i^c)$) which transform, respectively, as the fundamental and its conjugate representation of SU(3). ϕ_3^i and ϕ_{23}^i are scalar anti-triplet fields responsible for the symmetry breaking of SU(3) with H_α being the two Higgs doublets of the Minimal Supersymmetric Standard Model (MSSM), which are singlets under the SU(3) family symmetry. When the symmetry is spontaneously broken the scalar field ϕ_3 acquires a vev (vacuum expectation value) such that the term $\frac{1}{M_3^2}\psi_i \phi_3^i \psi_j^c \phi_3^j$ produces a Yukawa coupling only for the $(3,3)$ entry. The vev of ϕ_{23} produces entries $(2,2)$, $(2,3)$ and $(3,2)$ at order ε^2, through the second operator in eq. (2.7), where $\varepsilon = \langle \phi_{23} \rangle / M$. The other entries of eq. (2.6) are generated through higher dimensional operators.

The coefficients z, x, y, w and u are complex numbers of order 1 and can be fitted to reproduce the values of the atmospheric, solar and reactor neutrino mixing angles [1]. The coefficient a_f is a coefficient that depends on the nature of fermion coupling to the Higgs boson. This is described as an effective 120 or 126 SO(10) representation, coming from a coupling to H_{10} and Σ_{45}. Here Σ acquires a vev given by $\langle \Sigma \rangle = B - L + \kappa T_{R,3}$, where B and L are the baryon number and lepton number operators, respectively, and $T_{R,3}$ is the third component of isospin in SO(10). We have two solutions

$$\begin{aligned} \kappa &= 0 , \qquad a_l = -3 , \quad a_\nu = -3 , \\ \kappa &= 2 , \qquad a_l = +3 , \quad a_\nu = 0 \end{aligned} \tag{2.8}$$

but only with the last one we obtain large atmospheric and solar mixings [1].

Inverted hierarchies in the context of a $\mathrm{U}(1)_F$ symmetries $\mathrm{U}(1)_F$ family symmetries provide a convenient way of organizing the hierarchies within the Yukawa matrices for fermions (see for example [36, 37]). We consider here a $\mathrm{U}(1)_F$, which is broken by a set of singlets θ and $\bar{\theta}$, such that the breaking scale, M_Y, is set by $\theta = \bar{\theta}$, where the vevs are acquired along a 'D-flat direction'. The general idea is that at tree-level the $\mathrm{U}(1)_F$ symmetry

[2]The group SU(3) as a family symmetry has been broadly considered in the past [24]–[35].

only allows one Yukawa coupling, generally for the third family, due to the dominance of the top quark Yukawa coupling. Smaller Yukawa couplings may be generated effectively from higher dimension non-renormalizable operators. Such operators correspond to insertions of θ and $\bar{\theta}$, and hence to powers of the expansion parameter $\epsilon = \langle\theta\rangle/M_Y$. The power of the expansion parameter is controlled by the $\mathrm{U}(1)_F$ charges of the particular operator. The relevant fields for leptons are the lepton doublets L_i, the charge conjugated right-handed neutrinos and charged leptons $(\nu_R^o)^c$, $(l^o)^c$, the up-type Higgs doublet H_u and a single scalar field, H_M. The vev of the latter is responsible for giving mass to the heavy Majorana fields. We may denote the charges of these fields by, l_i, n_i, e_i, h_u and h_M respectively, and, within this convention, the Yukawa couplings for neutrinos and charged lepton may be written in the form

$$(Y^\nu)_{ij} = \epsilon^{|l_i+n_j|}, \qquad (Y^e)_{ij} = \epsilon^{|l_i+e_j|}, \tag{2.9}$$

where we have re-absorbed the charge h_u into the definition of the lepton charges l_i. The heavy right handed neutrino mass matrix is given by

$$(M_R)_{ij} = \epsilon^{|n_i+n_j+h_M|}\langle H_M\rangle\,. \tag{2.10}$$

The assignment of charges under the $\mathrm{U}(1)_F$ is constrained to reproduce lepton masses and mixings. The mass matrices for quarks can also be expressed in terms of $\mathrm{U}(1)_F$ symmetries, whose Yukawa couplings would be of the form $(Y^q)_{i,j} = \epsilon^{|(q_L)_i+(q_R)_j|}$, where q_i are the $\mathrm{U}(1)_F$ charges. However, as we have seen in [38], the kind of predictions for elements of the CKM matrix would be disfavoured, according to precision tests against the experimental measurements contributing to the CKM matrix. Nevertheless, if the $\mathrm{U}(1)_F$ symmetries are realized in the context of a GUT theory, it is possible to improve their predictions (see for example [39]).

3. U_{MNS} in the symmetry basis

The mixing matrices for quarks and leptons are described in terms of a unitary 3×3 matrix which in general can be parameterized in terms of 3 angles and 6 phases, namely,

$$\mathrm{diag}(e^{i\sigma_3}, e^{i\sigma_4}, e^{i\sigma_5})U\mathrm{diag}(e^{i\sigma_1}, 1, e^{i\sigma_2}) \equiv P'(\sigma)UP(\sigma) \tag{3.1}$$

where U contains a single phase, δ. For quarks the five σ_m phases may be absorbed into the re-definition of quark fields but for leptons, due to the Majorana nature of neutrinos, we may absorb just three phases into the re-definition of the charged lepton fields. Thus, we are left with *two of the* σ_m phases which may, in turn, be associated with the effective Majorana neutrino mass matrix, $(m_{LL}^\nu)_{\mathrm{diag}} = \mathrm{diag}(m_{\nu_1}e^{-2i\sigma_1}, m_{\nu_2}, m_{\nu_3}e^{-2i\sigma_2})$, or may remain in the mixing matrix:

$$U_{\mathrm{MNS}} = UP(\sigma)\,. \tag{3.2}$$

The latter convention, adopted here, for U_{MNS} requires three mixing angles and three CP violation phases. One of these phases is analogous to the case of quarks, δ, which is often called the *Dirac CP violation* phase, and the other two, σ_1 and σ_2, are the *Majorana CP* *violation* phases. For the case of leptons and quarks, the standard parameterization for U adopted here is:

$$\begin{aligned} U &= R_{23}P(-\delta,1,1)R_{13}P(\delta,1,1)R_{12} \\ &= \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}s_{12}c_{23}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned} \tag{3.3}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$. Here the mixing angles vary between 0 and $\pi/2$ and δ varies between 0 and 2π. Thus U may be expressed as the product of the matrices R_{ij}, rotations in the ij plane, such that $(R_{ij})_{ij} = s_{ij}$, and diagonal matrices with phases $P(\delta,1,1) = \mathrm{diag}(e^{i\delta},1,1)$. The mixing angle measured in atmospheric experiments is identified with θ_{23}, that measured in solar experiments with θ_{12} and that mixing angle measured in reactor experiments, with θ_{13}.

Given the matrices diagonalizing the mass matrices appearing in the effective mass lagrangian for leptons, eq. (2.2), and the definition of the effective neutrino mass matrix, eq. (2.3), such that[3]

$$m^l = L^l_m \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} R^{l\dagger}_m\,, \qquad m^\nu_{LL} = L^{\nu *}_m \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_3} \end{pmatrix} L^{\nu\dagger}_m\,, \tag{3.4}$$

then the mixing matrix U_{MNS}, relating mass eigenstates (ν_1,ν_2,ν_3) with states participating in neutrino oscillations, (ν_e,ν_μ,ν_τ),

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{3.5}$$

may be expressed by

$$U_{\mathrm{MNS}} = L^{l\dagger}_m L^\nu_m\,. \tag{3.6}$$

This combination $(L^{l\dagger}_m L^\nu_m)$ can be brought to the form of eq. (3.2), with U as in eq. (3.3).

In determining the relationship between the Dirac CP violation phase and the phases of the mass matrix, it is usually easier to work with the hermitean matrix $H = m^\dagger m$ rather than m itself. Of course the mixing angles are the same whether computed from the diagonalization of m or from that of H. If $m_{\mathrm{diag}} = L^\dagger_m m R_m$, where m_{diag} is a real diagonal matrix, then the hermitean matrix H is diagonalized by $m^2 = L^\dagger H L$, where L_m and L are related through a diagonal matrix of phases (see appendix A). Here we employ a diagonalization of H so that it may be applied for both canonical and inverted hierarchies, although in some cases the computation of the mixing angles from the hermitean matrix H is more tedious and obscures the simplicity of the relationship of the phases. In appendix A we detail the procedure of diagonalization; here we just note that since R and $L^\dagger$ are unitary matrices, they may also be parameterized in terms of three angles and six phases. Of these phases, only three: $\gamma^{f\prime}_{12}$, $\gamma^{f\prime}_{13}$ and γ^f_{23}; can be fixed by the elements of H and the others, α^ν_i,

[3] Note that in this convention $m^\nu_{\mathrm{diag}} = L^{\nu t}_m m^\nu_{LL} L^\nu_m = L^{\nu\dagger}_m m^{\nu\dagger}_{LL} L^{\nu *}_m$, for m^ν_{diag} real.

are used to fix the eigenvalues of m^{ν}_{LL} to be real. When constructing the $U_{\rm MNS}$ matrix from $L^{l\dagger}$ and L^{ν} the three undetermined phases, a^l_i, in L^l can be used to fix the three physical phases appearing there. Thus we can write the diagonalization matrices as follows

$$\begin{aligned}
L^f_m &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\gamma^{f\prime}_{13})} & 0 \\ 0 & 0 & e^{i(\gamma^f_{23}+\gamma^{f\prime}_{13})} \end{pmatrix} R^f_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\gamma^{f\prime}_{12}-\gamma^{f\prime}_{13})} & 0 \\ 0 & 0 & 1 \end{pmatrix} R^f_{13} R^f_{12} \begin{pmatrix} e^{-i\alpha_0} & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{pmatrix} \\
&= P^f_1 R^f_{23} P^f_2 R^f_{13} R^f_{12} P^f_3\,.
\end{aligned} \tag{3.7}$$

The super-script here refers to $f = l$, for charged leptons and to $f = \nu$, for neutrinos. In terms of the matrices in eq. (3.7), the mixing matrix $U_{\rm MNS}$ acquires the form

$$U_{\rm MNS} = P^{l\dagger}_3 R^{lt}_{12} R^{lt}_{13} P^{l\dagger}_2 R^f_{23} P^{l\dagger}_1 P^{\nu}_1 R^{\nu}_{23} P^{\nu}_2 R^{\nu}_{13} R^{\nu}_{12} P^{\nu}_3\,, \tag{3.8}$$

which we can express in terms of three angles and three physical phases by identifying each entry of $U_{\rm MNS}$ with the parameters appearing in the standard parameterization, eq. (3.3). A similar construction to this has been carried out in [40], we obtain:

$$\begin{aligned}
s_{13}e^{-i\delta_O} &= s^{\nu}_{13}c^l_{13}c^l_{12}e^{-i\delta_1} - c^{\nu}_{13}(s^l_{13}c^L_{23}c^l_{12}e^{-i\delta_2} - s^l_{12}s^L_{23}e^{-i\delta_3}) = (U_{\rm MNS})_{e3} \\
s_{12}c_{13} &= |s^{\nu}_{12}(c^{\nu}_{13}c^l_{13}c^l_{12} + s^{\nu}_{13}e^{i\delta_1}(c^l_{12}c^L_{23}s^l_{13}e^{-i\delta_2} + s^l_{12}s^L_{23})) + c^{\nu}_{12}s^l_{13}c^l_{12}s^L_{23}e^{-i\delta_2} \\
&\quad -c^{\nu}_{12}s^l_{12}c^L_{23}e^{-i\delta_3}| \\
c_{12}c_{13} &= |c^{\nu}_{12}(c^{\nu}_{13}c^l_{13}c^l_{12} + s^{\nu}_{13}e^{i\delta_1}(c^l_{12}c^L_{23}s^l_{13}e^{-i\delta_2} + s^l_{12}s^L_{23})) - s^{\nu}_{12}c^l_{12}s^l_{13}s^L_{23}e^{-i\delta_2} \\
&\quad +s^{\nu}_{12}s^l_{12}c^L_{23}e^{-i\delta_3}| \\
s_{23}c_{13} &= |s^L_{23}c^l_{12}c^{\nu}_{13}e^{-i\delta_3} - c^L_{23}c^{\nu}_{13}s^l_{12}s^l_{13}e^{-i\delta_2} + c^l_{13}s^l_{12}s^{\nu}_{13}e^{-i\delta_1}| \\
c_{23}c_{13} &= |c^L_{23}c^l_{13}c^{\nu}_{13}e^{-i\delta_2} + s^l_{13}s^{\nu}_{13}e^{-i\delta_1}|\,, \quad \text{for} \\
\theta_{23} &= \theta_{\rm atm}\,, \quad \theta_{12} = \theta_{\rm sol}\,, \quad \theta_{13} = \theta_{\rm rct}\,,
\end{aligned} \tag{3.9}$$

where

$$\begin{aligned}
\delta_1 &= \gamma^{\nu\prime}_{13} - \gamma^{\nu\prime}_{12} - (\xi_s - \xi_c) \\
\delta_3 &= (\gamma^{l\prime}_{12} - \gamma^{\nu\prime}_{12}) + (\chi - \xi_c)\,, \qquad \delta_2 = \gamma^{\nu\prime}_{12} + \gamma^{l\prime}_{13} - \gamma^{l\prime}_{12}\,, \qquad \chi = \gamma^l_{23} - \gamma^{\nu}_{23} \\
\xi_c &= \mathrm{Arg}(s^l_{23}s^{\nu}_{23} + c^l_{23}c^{\nu}_{23}e^{-i\chi})\,, \qquad \xi_s = \mathrm{Arg}(-s^l_{23}c^{\nu}_{23} + c^l_{23}s^{\nu}_{23}e^{-i\chi}) \\
c^L_{23} &= |s^{\nu}_{23}s^l_{23} + c^{\nu}_{23}c^l_{23}e^{i\chi}|\,, \qquad s^L_{23} = |c^l_{23}s^{\nu}_{23}e^{-i\chi} - s^l_{23}c^{\nu}_{23}|\,.
\end{aligned} \tag{3.10}$$

Using these formulas one can readily identify the following interesting points: (a) When we are working in the flavour basis, we can see that the Dirac CP violation phase, $\delta_{\rm O}$ is given by $\delta_1 = \gamma^{\nu\prime}_{13} - \gamma^{\nu\prime}_{12}$, as we have noted previously. (b) The mixing angle θ_{13} has contributions from θ^l_{12}, θ^{ν}_{13} and θ^l_{13} such that if these two last terms are negligible in comparison to the first one, we can relate the reactor angle to parameters of the charged lepton mass matrix [1]. (c) The angle θ_{23} has a term that goes like $\theta^{\nu}_{23} - \theta^l_{23}$, so even if the effective neutrino mass matrix in the symmetry basis is such that $t^{\nu}_{23} = 1$, there will be a deviation from maximality due to the charged leptons. The same happens in the case of θ_{12}, this is particularly relevant for models that predict θ^{ν}_{12} nearly maximal because the

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θ^l_{12} contribution can bring it down to the appropriate value, eq. (1.1). (*d*) There are some models that predict θ^ν_{13} to small or too big in comparison to the limit of eq. (1.1), however an appropriate contribution from s^l_{13} or s^l_{12} could agree with the limit.

The strength of CP violation in neutrino oscillations is expressed in terms of the invariant $\mathrm{Im}[(U_{\rm MNS})_{11}(U_{\rm MNS})_{22}(U_{\rm MNS})^*_{12}(U_{\rm MNS})^*_{21}]$ which can be written as

$$J_{\rm CP} = \frac{1}{8}\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\sin(\delta_{\rm O}) \tag{3.11}$$

In the flavour basis we can write this invariant in terms of the hermitean matrix $H^\nu = m^{\nu\dagger}m^\nu$ with phases γ^ν_{ij}, using eqs. (A.9), we have

$$\mathrm{J}_{\rm CP} = -\frac{\mathrm{Im}[(H^\nu_s)_{12}(H^\nu_s)_{23}(H^\nu_s)_{31}]}{\Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}}\,, \tag{3.12}$$

which can be used without constructing the $U_{\rm MNS}$ matrix as has been pointed out in [41].

4. CP violation phase from neutrino oscillations

4.1 Predictions of a class of hierarchical neutrino mass matrices

We assume that the low energy neutrinos acquire their mass through the see-saw mechanism, eq. (2.3). Here, let us write explicitly the form of the effective Majorana mass matrix $(m^\nu_{LL})^\dagger =$

$$\begin{pmatrix} \frac{m^{D2}_{12}}{M_2}+\frac{m^{D2}_{13}}{M_3} & \frac{m^D_{12}m^D_{22}}{M_2}+\frac{m^D_{13}m^D_{23}}{M_3} & \frac{m^D_{12}m^D_{32}}{M_2}+\frac{m^D_{13}m^D_{33}}{M_3} \\ \frac{m^D_{21}m^D_{22}}{M_2}+\frac{m^D_{13}m^D_{23}}{M_3} & \frac{m^{D2}_{21}}{M_1}+\frac{m^{D2}_{22}}{M_2}+\frac{m^{D2}_{23}}{M_3} & \frac{m^D_{21}m^D_{31}}{M_1}+\frac{m^D_{22}m^D_{32}}{M_2}+\frac{m^D_{23}m^D_{33}}{M_3} \\ \frac{m^D_{12}m_{32}}{M_2}+\frac{m^D_{13}m^D_{33}}{M_3} & \frac{m^D_{21}m^D_{31}}{M_1}+\frac{m^D_{22}m^D_{32}}{M_2}+\frac{m^D_{23}m^D_{33}}{M_3} & \frac{m^{D2}_{31}}{M_1}+\frac{m^{D2}_{32}}{M_2}+\frac{m^{D2}_{33}}{M_3} \end{pmatrix}, \tag{4.1}$$

where we have written $m^D = m^\nu_D$ for simplicity and have assumed that $m^\nu_{D11} = 0$ and $m^\nu_{\rm M} = \mathrm{diag}(M_1, M_2, M_3)$. Under the hierarchy

$$\frac{|m^\nu_{D31}|^2, |m^\nu_{D21}|^2, |m^\nu_{D21}m^\nu_{D31}|}{M_1} \gg \frac{m^\nu_{Di2}m^\nu_{Dj2}}{M_2} \gg \frac{m^\nu_{Di3}m^\nu_{Dj3}}{M_3}\,; \qquad i,j = 1,2,3\,; \tag{4.2}$$

which is often referred as *right-handed neutrino sub-sequential dominance* [37, 40] it is possible to explain large mixing angles for atmospheric and solar neutrinos and an small reactor neutrino mixings, see appendix A for the form of the mixing angles and masses. An specific realization of this pattern has been presented in [1], and we will discuss the implications for CP violation in neutrino oscillations in the next subsection. At the moment let us analyze, in this class of models, the determination of the CP violation phase in the symmetry basis. Note from eqs. (3.9), (3.10) that this is given in terms of the angles θ^ν_{13}, θ^ν_{12} and θ^ν_{23}, entering in the diagonalization of the effective neutrino mass matrix, the angles θ^l_{13}, θ^l_{12} and θ^l_{23}, entering in the diagonalization of the charged lepton matrix, and the phases $\gamma^{f\prime}_{12}$, $\gamma^{f\prime}_{13}$ and $\gamma^{f\prime}_{23}$ for $f = \nu, l$. Note that $\gamma^{\nu\prime}_{12}$, $\gamma^{\nu\prime}_{13}$ can be determined from eqs. (A.9), and γ_{23} can be obtained from eq. (A.2), which is equivalent to ask for a real tangent of the mixing angle θ_{23} -eq. (B.7),

$$\gamma^\nu_{23} = \phi^D_{31} - \phi^D_{21} \tag{4.3}$$

and, with this information and eq. (A.8), equivalent to the condition of having a positive real tangent t^{ν}_{12}, we have

$$
\begin{aligned}
c^{\nu}_{23}|m^{\nu}_{D22}|\sin(\xi_{22}) &= s^{\nu}_{23}|m^{\nu}_{D23}|\sin(\xi_{23})\,, \\
-\xi_{22} &= \phi^{D}_{12} - \phi^{D}_{22} + \gamma^{\nu\prime}_{12} \\
-\xi_{32} &= \phi^{D}_{12} - \phi^{D}_{32} + \gamma^{\nu}_{23} + \gamma^{\nu\prime}_{12}\,.
\end{aligned} \tag{4.4}
$$

Let us call $\delta^{\nu} = \gamma^{\nu\prime}_{13} - \gamma^{\nu\prime}_{12}$, a motivation for this definition is that in the limit of the flavour basis this combination is the Dirac CP violation phase in the lepton sector δ_{O}, eq. (3.10). Thus ξ_{22} and ξ_{23} can be rewritten as

$$
\xi_{22} = \phi^{D}_{22} - \phi^{D}_{12} - \gamma^{\nu\prime}_{13} + \delta^{\nu}\,, \qquad \xi_{32} = \phi^{D}_{32} - \phi^{D}_{31} + \phi^{D}_{21} - \phi^{D}_{12} - \gamma^{\nu\prime}_{13} + \delta^{\nu}\,. \tag{4.5}
$$

From eq. (A.9) we can determine $\gamma^{\nu\prime}_{13}$, to leading order, to obtain

$$
\begin{aligned}
\gamma^{\nu\prime}_{13} &= \phi^{D}_{21} - \phi^{D}_{12} - \eta_{12} \\
\eta_{12} &\equiv \mathrm{Arg}[m^{\nu *}_{D22} m^{\nu}_{D21} + m^{\nu *}_{D32} m^{\nu}_{D31}]
\end{aligned} \tag{4.6}
$$

inserting the expression for $\gamma^{\nu\prime}_{13}$ in eqs. (4.5), (4.4) we obtain

$$
\tan(\eta_{12} + \delta^{\nu}) \approx \frac{|m^{\nu}_{D22}|c^{\nu}_{23}\sin(\phi^{D}_{22} - \phi^{D}_{21}) - |m^{\nu}_{D32}|s_{23}\sin(\phi^{D}_{32} - \phi^{D}_{31})}{-|m^{\nu}_{D22}|c^{\nu}_{23}\cos(\phi^{D}_{22} - \phi^{D}_{21}) + |m^{\nu}_{D32}\cos(\phi^{D}_{32} - \phi^{D}_{31})|}\,. \tag{4.7}
$$

If we have $|m^{\nu}_{D22}c^{\nu}_{23}| = |m^{\nu}_{D32}s^{\nu}_{23}|$ then

$$
\begin{aligned}
\delta^{\nu} &\approx -2\eta_{12} - \frac{\pi}{2} \\
\eta_{12} &\approx \frac{(\phi^{D}_{21} - \phi^{D}_{22} + \phi^{D}_{31} - \phi^{D}_{32})}{2}\,.
\end{aligned} \tag{4.8}
$$

In the real case, as can seen from eq. (4.7), $\delta^{\nu} = 0$. In the limit in which $|m^{D}_{22}| = 0$, which could give also maximal mixing, we have

$$
\delta^{\nu} = -2\eta_{12} = (\phi_{32} - \phi_{31}) \tag{4.9}
$$

which agrees with the result presented in [2].

4.2 Predictions of a class of inverted hierarchical neutrino mass matrix

It is possible to obtain an inverted hierarchy $IH2$, eq. (2.5), describing the mass splittings and mixings of the low energy neutrinos, under the following conditions

$$
\begin{aligned}
\frac{m^{\nu}_{Di2} m^{\nu}_{Dj2}}{M_2} &\gg \frac{m^{\nu}_{Di3} m^{\nu}_{Dj3}}{M_3}\,; \quad i,j = 1,2,3\,; \\
\frac{|m^{\nu}_{D31}|^2, |m^{\nu}_{D21}|^2, |m^{\nu}_{D21} m^{\nu}_{D31}|}{M_1} &\gg \frac{m^{\nu}_{Dk2} m^{\nu}_{Dl2}}{M_2} \quad k,l = 2,3\,; \\
\frac{m^{\nu}_{D21} m^{\nu}_{D31}}{M_1} &= O\left(\frac{m^{\nu 2}_{12}}{M_2}\right).
\end{aligned} \tag{4.10}
$$

Using the procedure in appendix A, we can diagonalized the matrix $m^{\nu}_{\rm LL}$, with a diagonalization matrix L_m of the form eq. (A.16), except that now

$$\beta_1 = \gamma^{\nu}_{23} + \gamma^{\nu\prime}_{13} + \pi\,, \qquad \beta_2 = \gamma^{\nu\prime}_{13}\,, \qquad \beta_3 = \gamma^{\nu\prime}_{12} - \gamma^{\nu\prime}_{13}\,. \tag{4.11}$$

In this case the mixing angles are given approximately by

$$\begin{aligned}
t^{\nu}_{23} &\approx \frac{|m^{\nu}_{D31}|}{|m^{\nu}_{D21}|}\\
t^{\nu}_{13} &\approx \frac{|m^{\nu}_{D31}m^{\nu}_{D22} - m^{\nu}_{D21}m^{\nu}_{D32}|}{|m^{\nu}_{D12}|\sqrt{|m^{\nu}_{D21}|^2 + |m^{\nu}_{D31}|^2}}\\
t^{\nu}_{12} &\approx 1 - \frac{\Delta_{12}}{2}\\
\Delta_{12} &= \frac{\sqrt{|m^{\nu}_{D21}|^2 + |m^{\nu}_{D31}|^2}}{|m^{\nu}_{D12}|}\left(\frac{\frac{M_2}{M_1}(|m^{\nu}_{D21}|^2 + |m^{\nu}_{D31}|^2) - |m^{\nu}_{D12}|^2}{|m^{\nu}_{D22}m^{\nu *}_{D21} + m^{\nu}_{D32}m^{\nu *}_{D31}|}\right)\cos(\eta_{12})\\
\eta_{12} &\equiv {\rm Arg}[m^{\nu *}_{D22}m^{\nu}_{D21} + m^{\nu *}_{D32}m^{\nu}_{D31}]\,.
\end{aligned} \tag{4.12}$$

If the matrix $m^{\nu}_{\rm LL}$ is to be realized in the flavour basis, then in order for t^{ν}_{23} to account for the mixing of the atmospheric neutrinos, m^{ν}_{21} and m^{ν}_{31} need to be of the same order. From the expression of t^{ν}_{13} in eq. (4.12), we can see that in order for this mixing to describe the reactor experiments, there should be a cancellation between m^{ν}_{D22} and m^{ν}_{D32}. Finally $\frac{M_2}{M_1}(|m^{\nu}_{D21}|^2 + |m^{\nu}_{D31}|^2) - |m^{\nu}_{D12}|^2$ is constrained to reproduce Δ_{12} small, $\Delta_{12} \approx (0.21, 0.77)$, according to eq. (1.1), in order to account for the mixing of the solar neutrino experiments.

The masses of the low energy neutrinos are given approximately by

$$\begin{aligned}
m_{\nu_3} &\approx \frac{m^{\nu 2}_{D12}}{M_1}s^{\nu 2}_{13}\\
m_{\nu_2} &\approx e^{-2i\phi_{12}}\left(c^{\nu 2}_{12}m^{\nu\prime}_{11} + s^{\nu 2}_{12}e^{2i\gamma^{\prime}_{12}}m^{\nu\prime}_{22} + \frac{2c^{\nu}_{12}s^{\nu}_{12}}{M_2}\frac{(m^{\nu}_{D21}m^{\nu *}_{D22} + m^{\nu}_{D31}m^{\nu *}_{D32})}{\sqrt{|m^{\nu}_{D21}|^2 + |m^{\nu}_{D31}|^2}}\right)\\
m_{\nu_1} &\approx e^{-2i\phi_{12}}\left(s^{\nu 2}_{12}m^{\nu\prime}_{11} + c^{\nu 2}_{12}e^{2i\gamma^{\prime}_{12}}m^{\nu\prime}_{22} - \frac{2c^{\nu}_{12}s^{\nu}_{12}}{M_2}\frac{(m^{\nu}_{D21}m^{\nu *}_{D22} + m^{\nu}_{D31}m^{\nu *}_{D32})}{\sqrt{|m^{\nu}_{D21}|^2 + |m^{\nu}_{D31}|^2}}\right),
\end{aligned}$$

where

$$|m^{\nu\prime}_{22}| = \frac{|m^{\nu}_{D21}|^2 + |m^{\nu}_{D31}|^2}{|M_1|}\,, \qquad |m^{\nu\prime}_{11}| = \frac{|m^{\nu}_{D12}|^2}{|M_2|}\,. \tag{4.13}$$

Note from eq. (4.10) that the terms in eq. (4.13) multiplying $2c^{\nu}_{12}s^{\nu}_{12}$ are smaller than the terms multiplying $(c^{\nu}_{12})^2$ or $(s^{\nu}_{12})^2$, and since $|m^{\nu\prime}_{22}|$ and $|m^{\nu\prime}_{11}|$ need to be very close to each other in order to reproduce a small Δ_{12}, then $m_{\nu_1} \lesssim m_{\nu_2}$. The phases appearing in (4.13) can be computed from eqs. (A.9) or can be determined from the conditions of having real values for the tangents of the mixing angles. Thus we have

$$\begin{aligned}
\gamma^{\nu}_{23} &\approx \phi^{D}_{31} - \phi^{D}_{21}\\
\gamma^{\nu\prime}_{13} &= (\phi^{D}_{21} - \phi^{D}_{12}) - \eta_{12}\,.
\end{aligned} \tag{4.14}$$

We determine the phase $\delta^\nu = \gamma_{13}^{\nu\prime} - \gamma_{12}^{\nu\prime}$, as in section 4.1. From eq. (A.8), or from the condition of having a positive real tangent t_{12}^ν, we have that

$$\begin{aligned} |m_{22}^{\nu\prime}| \sin(\xi_{22}) &= |m_{11}^{\nu\prime}| \sin(\xi_{11}) \\ \xi_{22} &= \phi_{22}^{D\prime} - \phi_{12}^{D\prime} , \qquad \xi_{22} = \phi_{11}^{D\prime} - \phi_{12}^{D\prime} , \end{aligned} \tag{4.15}$$

where

$$\begin{aligned} \phi_{22}^{D\prime} &= 2(\gamma_{13}^{\nu\prime} - \phi_{21}^{D}) , \qquad \phi_{11}^{D\prime} = -2\phi_{12}^{D}, \\ \phi_{12}^{D\prime} &= \gamma_{13}^{\nu\prime} - \phi_{12}^{D} - \phi_{21}^{D} + \eta_{12}. \end{aligned} \tag{4.16}$$

Thus, inserting eq. (4.16) into eq. (4.15), we can write for δ^ν the following expression

$$\tan(\delta^\nu) = \frac{|m_{22}^{\nu\prime}| \sin(-2\eta_{12})}{|m_{22}^{\prime}| \cos(-2\eta_{12}) + |m_{11}^{\nu\prime}|} . \tag{4.17}$$

In the limit $|m_{22}^{\nu\prime}| = |m_{11}^{\nu\prime}|$, δ^ν is simply given by

$$\delta^\nu = -\eta_{12} = \mathrm{Arg}[m_{D22}^{\nu *} m_{D21}^{\nu} + m_{D32}^{\nu *} m_{D31}^{\nu}] . \tag{4.18}$$

4.3 Examples

4.3.1 Predictions for the texture inspired in a $\mathrm{SU}(3)_F \times \mathrm{SO}(10)$ symmetry

Here we consider matrices of the form of eq. (2.6), where we have mentioned that only the solution $\kappa = 2$, $a_l = +3$, $a_\nu = 0$ produces a large atmospheric and solar mixing compatible with experiments [1]. In this case

$$Y_{22}^\nu = \mathcal{O}(Y_{23}^\nu, Y_{12}^\nu, Y_{13}^\nu) \tag{4.19}$$

and with a diagonal matrix for right-handed neutrinos such that $M_1 \ll M_2 \ll M_3$, we have then an effective Majorana mass matrix m_{LL}^ν, given by eq. (2.3), which satisfies the conditions (4.2). In this case, therefore we can use the form of the phases $\gamma_{ij}^{\nu\prime}$ obtained in section 4.1.

Since we are working in the symmetry basis we need to include the mixings from the charged lepton sector. The matrix (2.6) for $f = l$ can also be diagonalized by the procedure detailed in appendix A, but in this case the diagonalization process produces small mixing angles. From eqs. (A.9) we can determine the relevant phases contributing to the U_{MNS} matrix

$$\gamma_{12}^{l\prime} \approx \gamma_{12}^{l} = \phi_{22}^{l} - \phi_{12}^{l} , \qquad \gamma_{13}^{l\prime} \approx \gamma_{13}^{l} - \gamma_{23}^{l} = \phi_{23}^{l} - \phi_{13}^{l} . \tag{4.20}$$

The lepton mixing angles are given by

$$s_{12}^l \approx \frac{|m_{12}^l + \frac{m_{13}^l m_{23}^l}{m_{33}^l}|}{|m_{22}^l + \frac{m_{23}^{l2}}{m_{33}^l}|} = \mathcal{O}(\bar\epsilon) , \quad s_{23}^l \approx \frac{|m_{23}^l + \frac{m_{32}^l m_{22}^l}{m_{33}^l}|}{|m_{33}^l|} , \quad s_{13}^l \approx \frac{|m_{13}^l - \frac{m_{12}^l m_{23}^l}{m_{33}^l}|}{|m_{33}^l|} = \mathcal{O}(\bar\epsilon^3) . \tag{4.21}$$

With this information and the formulas appearing in eq. (3.9) we can see that the the solar and atmospheric mixing angles are mainly given by the angles θ^ν_{12} and θ^ν_{23}, respectively, appearing in the diagonalization of the neutrino mass matrix (4.1):

$$\tan\theta_{\rm sol} = |t_{12}| \approx |t^\nu_{12}| \left| \frac{1 - \frac{s^l_{12} c^L_{23} e^{-i\delta_3}}{t^\nu_{12}}}{1 + s^l_{12} t^\nu_{12} c^L_{23} e^{-i\delta_3}} \right|$$

$$\tan\theta_{\rm atm} = |t_{23}| \approx |t^L_{23}| \left| \frac{1 - \frac{s^l_{12} s^\nu_{13}}{t^L_{23}} e^{-i(\delta_1 - \delta_3)}}{1 + \frac{s^l_{13} s^\nu_{13}}{c^L_{23}} e^{-i(\delta_1 - \delta_2)}} \right|$$

$$c^L_{23} \approx c^\nu_{23} |1 + 2 t^\nu_{23} \theta^l_{23} \cos\chi| \,, \qquad s^L_{23} \approx s^\nu_{23} |1 - 2 t^{\nu\,-1}_{23} \theta^l_{23} \cos\chi| \,, \tag{4.22}$$

where the phases are given in eqs. (3.10). Note however that the mixing angle explained by reactor experiments, θ_{13}, can receive important contributions from θ^l_{12}, a mixing angle entering in the diagonalization of the charged lepton mass matrix, eq. (2.6), thus we have

$$\sin\theta_{\rm rct} e^{-i\delta_{\rm O}} = s_{13} e^{-i\delta_{\rm O}} \approx \theta^\nu_{13} e^{-i\delta_1} + \theta^l_{12} \sin^L_{23} e^{-i\delta_3} \,. \tag{4.23}$$

In order to determine $\delta_{\rm O}$ in this case we need to see which are the dominant terms in eq. (3.9). We consider here three such cases, $s^\nu_{13} \gg s^l_{12}$, $s^\nu_{13} \ll s^l_{12}$ and $s^\nu_{13} = \mathcal{O}(s^l_{12})$. These cases would correspond to *different* models for which the contributions from the charged leptons to the mixing matrix $U_{\rm MNS}$ are more or less important, and hence the results *are not* equivalent.

(*a*) $\mathbf{s^\nu_{13} \gg s^l_{12}}$. In this case the CP violation phase appearing in neutrino oscillations would be simply given by

$$\delta_{\rm O} \approx \delta_3 \approx \gamma^{\nu\prime}_{13} - \gamma^{\nu\prime}_{12} = \delta^\nu \,, \tag{4.24}$$

where δ^ν is determined by eq. (4.7).

(*b*) $\mathbf{s^\nu_{13} \ll s^l_{12}}$. In this case the dominant term in eq. (3.9) is s^l_{12} and hence

$$\delta_{\rm O} \approx \delta_3 = (\gamma^{l\prime}_{12} - \gamma^{\nu\prime}_{12}) + (\chi_c - \xi) \,. \tag{4.25}$$

From eqs. (3.10) we can see that for this case

$$\chi - \xi_c \approx \theta^l_{23} \sin\chi t^\nu \,, \qquad \gamma^{l\prime}_{12} \approx \gamma^l_{12} \approx \phi^l_{22} - \phi^l_{12} \tag{4.26}$$

and using eqs. (4.6) we have

$$\delta_{\rm O} \approx \gamma^l_{12} + \eta_{12} + \delta^\nu + (\phi^D_{12} - \phi^D_{21}) - \theta^l_{23} \sin\chi_c t^\nu_{23} \,. \tag{4.27}$$

In general δ^ν can be determined from eq. (4.7), let us take a particular case: $|m^\nu_{D22} c^\nu_{23}| = |m^\nu_{D32} s^\nu_{23}|$, so we can use eq. (4.8) and hence

$$\delta_{\rm O} \approx (\gamma^l_{12} - \eta_{12}) - \frac{\pi}{2} + (\phi^D_{12} - \phi^D_{21}) - \theta^l_{23} \sin\chi t^\nu_{23} \,. \tag{4.28}$$

For the particular choice of assigning a phase ϕ to the element (12) of $m^\nu_{\rm D}$ and a phase ϕ' to the element 13 of m^l we have $\delta_{\rm O} \approx -\frac{\phi}{2} - \theta^l_{23} \sin\phi'$, as we have seen in [1].

(*c*) $\mathbf{s^\nu_{13} \approx s^l_{12} \sin^L_{23}}$. In this case

$$s_{13} e^{-i\delta_{\rm O}} \approx \theta^\nu_{13} e^{-i\delta_1} + \theta^l_{12} \sin^L_{23} e^{-i\delta_3}\,, \tag{4.29}$$

thus we have

$$\delta_{\rm O} \approx \frac{\delta_1 + \delta_3}{2} = \delta^\nu + \frac{\eta_{12}}{2} + \frac{\gamma^{l\prime}_{12}}{2} + \frac{\phi^D_{12} - \phi^D_{21}}{2} - \frac{\theta^l_{23} \sin\chi}{2}\,. \tag{4.30}$$

For $|m^\nu_{D22} c^\nu_{23}| = |m^\nu_{D32} s^\nu_{23}|$, using eq. (4.8),

$$\delta_{\rm O} \approx -\frac{3}{2}\eta_{12} + \frac{\gamma^l_{12}}{2} + \frac{\phi^D_{12} - \phi^D_{21}}{2} - \frac{\theta^l_{23} \sin\chi}{2} - \frac{\pi}{2}\,. \tag{4.31}$$

Let us comment on a particular set of values of the mixing angles. Suppose that

$$s^l_{12} \approx \sqrt{\frac{m_e}{m_\mu}} \approx 0.07\,, \tag{4.32}$$

which can be obtained using them mass matrix parameterization of eq. (2.6), for charged leptons. For the case of the effective matrix for neutrinos, eq. (4.1), with the conditions of eq. (4.2), the neutrino mixing angle θ^ν_{13} is given by

$$t^\nu_{13} \approx \frac{M_1}{M_2} s^\nu_{23} \frac{|m^{D*}_{12} m^D_{22} + m^D_{13} m^{D*}_{23}|}{|m^D_{12}|^2 + |m^D_{13}|^2} \tag{4.33}$$

and the ratio M_1/M_2 is proportional to $r_\Delta \equiv \sqrt{\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}}$ due to the constraints on the masses of the neutrinos (see appendix A). For the particular realization of eq. (2.6) with a solution reproducing LMA angle, [1], we have $M_1/M_2 \approx r_\Delta/3$. The latest results of KamLAND [6] present two valid regions for Δm^2_{12} at 3σ, as a consequence we have two allowed regions for r_Δ:

$$\begin{aligned} \Delta m^2_{12} &\in [5.1, 9.7] \times 10^{-5}\,{\rm eV}^2 \to r_\Delta = 0.18^{-0.08}_{+0.11} \\ \Delta m^2_{12} &\in [1.2, 1.9] \times 10^{-4}\,{\rm eV}^2 \to r_\Delta = 0.26^{-0.10}_{+0.14}. \end{aligned} \tag{4.34}$$

The first region is the one that contains the best fit point (BFP) for $\Delta m^2_{12} = 6.9 \times 10^{-5}{\rm eV}^2$ and, within the 3σ region, s^ν_{13} can acquire values of order 10^{-2} which is one order of magnitude less than s^l_{12}, eq. (4.32). However the BFP for $r_\Delta = 0.16$ gives a value of $t^\nu_{13} = 0.05$ which is close to the value of $s^l_{12} s^\nu_{23} \approx 0.07$; thus the preferred solution for the BFP of Δm^2_{12} points out to the third of the cases presented above, **(c)**, and hence the prediction for the CP violation phase would be close to (4.30). In this case $\theta_{\rm rct} = O(10^{-1})$, which agrees with the latest bounds [11, 40].

Lepton flavour violating processes constraints. The model presented in this section satisfies the constraints from the lepton flavour violating processes (LFV), $\tau \to \mu\gamma$, $\mu \to e\gamma$. The branching ratios of these LFV, $B(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$ and $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$, depend on the Yukawa couplings for neutrinos and the heavy right-handed Majorana masses through the matrix [42]

$$C = Y^\nu {\rm Ln}\left(\frac{M_X}{M_R}\right) Y^{\nu\dagger}\,, \tag{4.35}$$

thus the bounds on the branching ratios can be translated in terms of bounds for the elements $C_{\tau\mu}$ and $C_{\mu e}$, respectively. These elements depend also on the region of the spectra for supersymmetric particles. Small values of $(C_{\tau\mu}, C_{\mu e}) \sim (10^{-1}, 10^{-3})$, correspond to light susy particles ($\sim 200\,\mathrm{GeV}$) and large values, $(C_{\tau\mu}, C_{\mu e}) \sim (10^2, 10^{-2})$, correspond to heavy susy particles ($\sim 800\,\mathrm{GeV}$) [42]. In this model we have $C_{\tau\mu} \approx 1$ and $C_{\mu e} \approx 10^{-3}$ hence this model would be possible for a light supersymmetric spectra. In [43] the LFV constraints have been analyzed for natural neutrino mass hierarchies.

Renormalization group equations effects. To express the mass splittings and phases at the electro-weak scale, M_{EW}, the evolution of the renormalization group equations (RGE's) from the GUT scale M_{G} down to M_{EW} has to be taken into account. In the context of the see-saw mechanism, due to the decay of the right-handed Majorana states at different scales, it is necessary to decouple these singlets at those scales and then consider the appropriate effective theories below them. The RGE's of the leptonic sector have the general form [44]

$$16\pi^2 \frac{d}{dt} X_i = F_{X_i}(X_i, X_j, \ldots)\,, \tag{4.36}$$

where $t = \ln(\mu/\mu_0)$, μ is the running scale, $X_i \in \{Y^\nu, M_R, Y^l, \ldots\}$ and F_{X_i} is the function describing the evolution of X_i. To account for these RGE's effects, one begins with the initial conditions and the coupled differential equations, eqs. (4.36), at M_{G} and then evolve them down to the scale, M_3, at which the heaviest R-H Majorana neutrino decouples. At this point the appropriate RGE's for the effective theory need to replace those considered at the GUT scale and then it is necessary to perform appropriate matching conditions and then continue this process until the scale of the decoupling of the lightest R-H Majorana state, M_1, is reached. At this scale the RGE's describing the evolution of the effective five dimensional operator producing the see-saw, eq. (2.3), can be used. This RGE has the form [44, 45, 46] $16\pi^2 dm^\nu_{LL}/dt = \alpha m^\nu_{LL} + P^T m^\nu_{LL} + m^\nu_{LL} P$ where $P = CY^l Y^{l\dagger}$, $C = 1, -3/2$ in the MSSM and the SM [47] respectively and α is a function of the Yukawa and gauge couplings. In order to account in a quantitatively accurate way for these effects the numerical evolution of the RGE's should be used, however qualitatively and, to a reasonable accuracy it is useful to employ analytical formulae for the running of masses, mixing angles and phases. In our analysis we have employed the results of [48] were the authors have derived analytic formulas for the running of the neutrino mixing angles $\theta^\nu_{i,j}$, the mass eigenvalues m_{ν_i}, the CP violating phase δ^ν_i and the Majorana phases $\sigma_{1,2}$. We find that for the case analyzed in this section the effects of the RGE's in the mixing angles is less than 7%, for the mass values is less than 5% and for the phases is also less than 5%.

4.3.2 An example of inverted hierarchy IH2

An explicit realization of a hierarchy $IH2$, eq. (2.5), satisfying the conditions (4.10), can be obtained with the following matrices

$$Y^\nu = \frac{m^\nu_D}{|m^\nu_{D33}|} = \begin{pmatrix} 0 & a_{12}\lambda_2 & a_{13}\lambda_2 \\ a_{21}\lambda_1 & a_{22}\lambda_2\epsilon & a_{32}\lambda_2\epsilon \\ a_{31}\lambda_1 & a_{32}\lambda_2\epsilon & a_{33}\lambda_2\epsilon \end{pmatrix}, \qquad m_M = \begin{pmatrix} M\lambda_1^2 & & \\ & M\lambda_2^2 & \\ & & M \end{pmatrix}, \tag{4.37}$$

where a_{ij} are complex coefficients of order 1 and $\epsilon, \lambda_i \ll 1$. We can see that it is possible to have t^{ν}_{23} of order 1 since at leading order it is simply given by $t^{\nu}_{23} = |a_{31}|/|a_{21}|$. The value of t^{ν}_{13} is naturally small because it is given at order ϵ

$$t^{\nu}_{13} = \frac{|a_{31}a_{22} - a_{32}a_{21}|}{|a_{12}|\sqrt{|a_{21}|^2 + |a_{31}|^2}}\epsilon\,. \tag{4.38}$$

If this structure is to be realized in the flavour basis then to have an acceptable value for Δ_{12}, eq. (4.12) we need

$$(|a_{21}|^2 + |a_{31}|^2 - |a_{12}|^2) = O(\epsilon)\,, \tag{4.39}$$

if it is realized in the symmetry basis, then $t_{\rm sol}$ can have contributions from the charged lepton sector (3.9) and it is possible to relax the condition (4.39). For an inverted hierarchy we have

$$\begin{aligned} \Delta m^2_{\rm atm} &= m_1^2 - m_3^2 \approx m_1^2\,, \qquad \text{for} \qquad |m_3| \ll |m_1| \\ \Delta m^2_{\rm sol} &= m_2^2 - m_1^2 \\ |m_{\nu_2}| &= O(10^{-2})\,\mathrm{eV}\,, \end{aligned} \tag{4.40}$$

thus $\Delta m^2_{\rm atm}$, eq. (1.1), fixes the value of M to be of order 10^{15} GeV and the value of $\Delta m^2_{\rm sol}$ and the order of the mass of the heaviest low energy left handed neutrino, ν_2, fix the order of ϵ to be $\lesssim \mathcal{O}(10^{-2})$.

The values of λ_1 and λ_2 are restricted to satisfy LFV bounds, to this end let us write the coefficients $C_{\mu e}$ and $C_{\tau\mu}$, described in eq. (4.35):

$$\begin{aligned} C_{\mu e} &= a_{22}a^*_{12}\lambda_2^2\epsilon\ln\left(\frac{M_X}{M\lambda_2^2}\right) + a_{23}a^*_{13}\lambda_2^2\epsilon\ln\left(\frac{M_X}{M}\right) \\ C_{\tau\mu} &= a_{31}a^*_{21}\lambda_1^2\ln\left(\frac{M_X}{M\lambda_1^2}\right) + \epsilon^2\left(\epsilon^2 a_{32}a^*_{22}\lambda_2^2\ln\left(\frac{M_X}{M\lambda_2^2}\right) + a_{33}a^*_{23}\lambda_2^2\ln\left(\frac{M_X}{M}\right)\right) \end{aligned} \tag{4.41}$$

Here we can consider two cases: (i) $\lambda_1 < \lambda_2$ and (ii) $\lambda_1 \approx \lambda_2$, both can explain the mixings observed by neutrino oscillation experiments but would have different behaviours for leptogenesis and could be explained by different symmetries.

(i) In this case for the values

$$\epsilon = 4\times 10^{-3}\,, \qquad \lambda_1 = 5\times 10^{-2}\,, \qquad \lambda_2 = 2.5\times 10^{-1} \tag{4.42}$$

and the coefficients

$$(|a_{ij}|) = \begin{pmatrix} 0 & 1.84 & 1.2 \\ 1.2 & 0.7 & 0.8 \\ 1.4 & 1.5 & 1 \end{pmatrix}, \qquad (\phi_{ij}) = \begin{pmatrix} 0 & 0 & 1.4 \\ 1.8 & 0.2 & 0.4 \\ 0.5 & 0.01 & 0.1 \end{pmatrix}, \tag{4.43}$$

where we have written $a_{ij} = |a_{ij}|e^{i\phi_{ij}}$,[4] we have

$$\Delta m^2_{13} = 4\times 10^{-3}\,\mathrm{eV}^2 \qquad \Delta m^2_{21} = 3.3\times 10^{-5}\,\mathrm{eV}^2$$

[4]The change in the coefficients does not change significatively the results. The only coefficient that is restricted is a_{12} because it has to satisfy the condition of eq. (4.39).

$$
\begin{aligned}
(t^\nu_{23})^2 &= 1.36 \qquad (t^\nu_{12})^2 = 0.69 \\
(t^\nu_{13})^2 &\approx 7\times 10^{-6} \\
M_1 &= 3.8\times 10^{12}\,\mathrm{GeV} \qquad M_2 = 6.1\times 10^{13}\,\mathrm{GeV} \\
M &= 1.05\times 10^{15}\,\mathrm{GeV} \\
m_{\nu_2} &= 0.084\mathrm{eV} \qquad m_{\nu_3} = 3\times 10^{-5}\mathrm{eV} \\
C_{\tau\mu} &= 0.052 \qquad C_{\mu e} = 0.0024\,.
\end{aligned}
\tag{4.44}
$$

Where we have given the values obtained at electro-weak scale, using the approximate RGE's formulas of [48]. In this case, the effects of the RGE's, for the SM or small $\tan\beta$ of the MSSM, is an increase up to 20% for $(t^\nu_{12})^2$, 1% for $(t^\nu_{23})^2$ and negligible for $(t^\nu_{13})^2$ and m_{ν_3}. This behaviour of $(t^\nu_{13})^2$ and m_{ν_3} corresponds to their smallness, compared to the other parameters, which is in turn produced by the small value of $\epsilon \approx 0.004$. For Δm^2_{21} there is an increase up to 60% and for Δm^2_{32} up to 50%. Larger values of $\tan\beta$ correspond to a larger increase, if $\tan\beta\approx 20$ then there is an increase of about 90% in $(t^\nu_{12})^2$, which brings it outside the valid experimental region to account for the solar neutrino oscillation experiments, eq. (1.1). The values presented in eq. (4.44) correspond to $\tan\beta = 6$. The effect of RGE's on the CP Dirac violating phase depends on the Majorana phases, we discuss this effect in section 6.

In this case we note that $\epsilon\approx\lambda_2^4$ and $\lambda_1\approx\lambda_2^2$ so we could write the m^ν_D in terms of a single parameter $\lambda=\lambda_2$, which is of the order of the Cabibbo angle $\theta_C\approx 0.22$. In this case we have

$$
Y^\nu = \begin{pmatrix} 0 & \lambda & \lambda \\ \lambda^2 & \lambda^5 & \lambda^5 \\ \lambda^2 & \lambda^5 & y_{33} \end{pmatrix}, \qquad M_R = \begin{pmatrix} M\lambda^4 & & \\ & M\lambda^2 & \\ & & M \end{pmatrix}, \tag{4.45}
$$

where y_{33} could be λ or λ^5. This symmetry can be considered in terms of a $\mathrm{U}(1)_F$ symmetry for the Yukawa couplings for leptons, in terms of eqs. (2.9), and for the Majorana mass matrix, given by eq. (2.10). Let us consider the following assignment of charges

$$
n_{1,2,3} = 2,1,0 \qquad l_{1,2,3} = 1,-5,-5 \qquad e_{1,2,3} = 11,3,5\,, \tag{4.46}
$$

then we have

$$
Y^\nu = \begin{pmatrix} \lambda^3 & \lambda^2 & \lambda \\ \lambda^3 & \lambda^4 & \lambda^5 \\ \lambda^3 & \lambda^4 & \lambda^5 \end{pmatrix}, \quad Y^e = \begin{pmatrix} \lambda^{12} & \lambda^4 & \lambda^6 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^6 & 1 & 1 \end{pmatrix}, \quad M_R = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}. \tag{4.47}
$$

With appropriate coefficients for Y^l it is possible to produce the eigenvalues proportional to the masses of the charged leptons and also small mixings for charged leptons:

$$
\begin{aligned}
(y_e, y_\mu, y_\tau) &\propto (\lambda^6, \lambda^2, 1) \\
s^l_{23} &= \mathcal{O}(10^{-1})\,, \qquad s^l_{13} = \mathcal{O}(\lambda^6)\,, \qquad s^l_{12} = \mathcal{O}(\lambda^2)\,.
\end{aligned}
\tag{4.48}
$$

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It is also possible to reproduce $m^\nu_{LL}(IH2)$ of the form eq. (2.5). We can diagonalize M_R with a matrix, L_M, of small mixings such that with $(M_R)_{\rm diag} = L^t_M M_R L_M$ and then we have

$$Y^{\nu\prime} = Y^\nu L_M = \begin{pmatrix} a'_{1,1}\lambda^3 & a'_{1,2}\lambda^2 & a'_{1,3}\lambda \\ a'_{2,1}\lambda^3 & a'_{2,2}\lambda^4 & a'_{2,3}\lambda^5 \\ a'_{3,1}\lambda^3 & a'_{3,2}\lambda^4 & a'_{3,3}\lambda^5 \end{pmatrix}, \qquad (M_R)_{\rm diag} \approx \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix} \langle H_{\rm M}\rangle\,, \tag{4.49}$$

which we can identify with eqs. (4.45), although we would need a cancellation for the element a'_{11} and the coefficients would need to reproduce the order of the power of λ as in eq. (4.45).

In this case, the atmospheric and mixing angles will be dominated by the mixings coming from the effective Majorana mass matrix, as in eq. (4.22), since s^l_{23} and s^l_{12} are small, eq. (4.48), but as we can observe from eq. (3.9), the reactor angle would be driven by $s^l_{12} = \mathcal{O}(\lambda^2)$, since $s^l_{13} = \mathcal{O}(\lambda^6)$ and $s^\nu_{13} = \mathcal{O}(\lambda^4)$. Thus we have

$$\sin\theta_{\rm rct} e^{-i\delta_{\rm O}} \approx \theta^l_{12} \sin^L_{23} e^{-i\delta_3}\,, \qquad \delta_{\rm O} = \delta_3 \approx \gamma^{l\prime}_{12} - \gamma^{\nu\prime}_{12}\,, \tag{4.50}$$

where $\gamma^{\nu\prime}_{12}$ and $\gamma^{\nu\prime}_{13}$ are given by eqs. (4.14) and thus

$$\delta_{\rm O} \approx \gamma^{l\prime}_{12} + (\delta^\nu + \eta_{12}) + (\phi^D_{21} - \phi^D_{12})\,, \tag{4.51}$$

and for the case of $|m^{\nu\prime}_{22}| = |m^{\nu\prime}_{11}|$, we have

$$\delta_{\rm O} \approx \gamma^{l\prime}_{12} + (\phi^D_{21} - \phi^D_{12})\,, \tag{4.52}$$

where we have used eq. (4.18).

(ii) In this case for the values

$$\epsilon = 3\times 10^{-2}\,, \qquad \lambda_1 = 1.2\times 10^{-1}\,, \qquad \lambda_2 = 1.4\times 10^{-1} \tag{4.53}$$

and the coefficients as in eq. (4.43), we have

$$\begin{aligned} \Delta m^2_{13} &= 1.3\times 10^{-3}{\rm eV}^2 \qquad \Delta m^2_{21} = 8.4\times 10^{-5}{\rm eV}^2 \\ (t^\nu_{23})^2 &= 1.35 \qquad (t^\nu_{12})^2 = 0.71 \qquad (t^\nu_{13})^2 \approx 2\times 10^{-4} \\ M_1 &= 3.7\times 10^{13}{\rm GeV} \qquad M_2 = 5\times 10^{13}{\rm GeV} \qquad M = 2.5\times 10^{15}{\rm GeV} \\ m_{\nu_2} &= 0.038{\rm eV} \qquad m_{\nu_3} = 0.003{\rm eV} \\ C_{\tau\mu} &= 0.15 \qquad C_{\mu e} = 0.006\,. \end{aligned} \tag{4.54}$$

The effects of the RGE's in the parameters at $M_{\rm EW}$ for the SM or small $\tan\beta$ of the MSSM, is similar to the previous case, except that the increase of $(t^\nu_{12})^2$ is more moderate, up to 15%. In this case due to the tendency for Δm^2_{12} to lie in the upper part of the allowed experimental region, see eq. (1.1), large $\tan\beta$ values (≥ 20) bring

Δm^2_{12} up to $\mathcal{O}(10^{-4})\mathrm{eV}^2$, outside the valid experimental region to account for solar neutrino experiments. For this example we note that $\lambda_1^2 \approx 2\epsilon$ then we would have

$$Y^\nu = \begin{pmatrix} 0 & \lambda_2 & \lambda_2 \\ \lambda_1 & 4\lambda_1^3 & 4\lambda_1^3 \\ \lambda_1 & 4\lambda_1^3 & y_{33} \end{pmatrix} \tag{4.55}$$

where y_{33} could be λ_2 or $4\lambda_1^4$. It is difficult to motivate this pattern in the context of $\mathrm{U}(1)_F$ symmetries, firstly because it requires two parameters and secondly because of the powers appearing in each entry.

5. CP violation in leptogenesis and its connection to neutrino oscillations

5.1 CP asymmetries from leptogenesis

According to cosmic microwave background radiation measurements, the observed abundance of the light elements synthesized during the Big Bang nucleosynthesis requires that the baryon asymmetry of the universe (BAU), parameterized by the baryon-to-entropy ratio, $Y_B = n_B/s$, lies in the range [49]

$$Y_B \in (0.7, 1) \times 10^{-10}\,. \tag{5.1}$$

In the leptogenesis scenario [50], a $B-L$ asymmetry is produced from the decay of the heavy right-handed Majorana neutrinos, N_j, which violate the lepton number at a large scale beyond the electroweak scale. The initially produced lepton asymmetry, Y_L, is converted into a net baryon asymmetry Y_B, through the $(B+L)$-violating sphaleron processes, such that at the end of the processes, Y_B and Y_L are related by

$$Y_B = \frac{\alpha}{\alpha-1} Y_L\,, \qquad \alpha = \frac{8N_F + 4N_H}{22N_F + 13N_H}\,, \tag{5.2}$$

where N_F is the number of families of heavy right handed neutrinos and N_H the number of Higgs multiplets. In thermal leptogenesis the right-handed neutrino number densities Y_{N_j} and the generated lepton asymmetry Y_L evolve with time according to a set of Boltzmann equations which depend on the physical processes occurring in the thermal bath and on the expansion of the universe. Here we assume the standard hot big bang universe, which is equivalent to assume a very high reheating temperature after inflation, larger than the right-handed neutrino masses, M_j. In the MSSM extended with heavy right-handed neutrinos the physical processes relevant to the generation of BAU are typically the decays and inverse decays of N_i and its scalar partners, $\widetilde{N^c}_i$, and L violating processes mediated by virtual N_i or $\widetilde{N^c}_i$ particles. Right handed neutrinos, N_i, decay into Higgs bosons and leptons or into Higgsinos and s-leptons; while $\widetilde{N^c}_i$ decay into Higgs bosons and s-leptons or into Higgsinos and leptons. In the SM, the correspondent physical processes take place. The CP asymmetries in the different decay channels of N_j and $\widetilde{N^c}_j$ can all be expressed by the same CP violation parameter ϵ_j [51],

$$\varepsilon_j = \frac{\Gamma(N_{R_j} \to l H_2) - \Gamma(N^\dagger_{R_j} \to l^\dagger H_2^\dagger)}{\Gamma(N_{R_j} \to l H_2) + \Gamma(N^\dagger_{R_j} \to l^\dagger H_2^\dagger)}$$

$$\varepsilon_j \approx -\frac{1}{8\pi (m_{fD}^{\nu\dagger} m_{fD}^{\nu})_{11} v^2} \sum_{i \neq j} \mathrm{Im}\left[(m_{fD}^{\nu\dagger} m_{fD}^{\nu})_{ji}^2\right] f\left(\frac{M_i^2}{M_j^2}\right), \tag{5.3}$$

where $f(x) = \sqrt{x}\,|(1+x)\ln(x/1+x) + (2-x)/(1-x)|$, m_D^ν is the Dirac matrix for neutrinos, eq. (2.1), and v_2 is the vacuum expectation of the Higgs field, H_2. The CP asymmetries, ϵ_j, are constrained to reproduce the observed value of Y_B, eq. (5.1).

In the case of hierarchical heavy neutrinos, the sign of Y_B is fixed by the sign of the CP asymmetry generated in the decay of the lightest heavy neutrino, ε_1. Such that for Y_B to be positive, as required by the observations, it is necessary to have $\varepsilon_1 < 0$. Given the present measurements of neutrino masses and oscillations, it appears plausible to associate the baryon number of the universe with the violation of lepton number. In this context it makes sense to determine if there is a correlation between the sign of the baryon number of the universe and the strength of CP violation in neutrino oscillations [4]. This correlation is relevant to look for in successful schemes that explain or accommodate the correct values of masses and mixings. As we can see from (5.3) this correlation will depend as well in which range we consider for the values M_i/M_j. We consider here the case $M_1 < M_2 \ll M_3$. In this case, there are simplifications in the treatment of the terms that enter in ϵ_j. For $M_1 < M_2 \ll M_3$, $f(x) \approx -\frac{3}{2\sqrt{x}}$, $x \gtrsim 15$ [52, 53, 40], then the only relevant CP asymmetry is the one produced by the lightest right handed neutrino, which can be expressed by

$$\varepsilon_1 \approx -\frac{3}{16\pi (m_{fD}^{\nu\dagger} m_{fD}^{\nu})_{11} v^2} \sum_{i \neq 1} \mathrm{Im}\left[(m_{fD}^{\nu\dagger} m_{fD}^{\nu})_{1i}^2\right] \frac{M_1}{M_i}, \tag{5.4}$$

this expression is given in the basis where both the charged leptons and the heavy right-handed neutrinos are diagonal.

In the context of thermal leptogenesis, when the observed baryon asymmetry is generated through the decays of the lightest heavy Majorana neutrino N_1, in order to produce the right baryon asymmetry, Y_B, there exists an upper bound on the lightest M_1, typically $M_1 \gtrsim 10^8$GeV [5]. Thermal leptogenesis requires that the reheating temperature T_R be such that $M_1 \lesssim T_R$, in this context a model independent analysis [54] has given a constraint of the re-heating temperature to be $T_R \approx M_1 = \mathcal{O}(10^{10}\mathrm{GeV})$. This temperature is marginally compatible with the maximum allowed one in supergravity theories, usually $T_R \lesssim (10^8 - 10^9)$GeV, which is usually constrained by thermal gravitino production. Thus we have a slight incompatibility between the reheating temperature T_R required by thermal leptogenesis and the one required by many supergravity theories. There are many options to overcome this problem. For example, it is possible to consider, still in the context of thermal leptogenesis, the decays of two heavy neutrinos which are quasi-degenerated in mass $M_1 \approx M_2$, [53]. In this case, the CP asymmetries ε_j are enhanced due to self-energy contributions and the required baryon asymmetry can be produced by right-handed heavy neutrinos with masses $M_1 \approx M_2 \lesssim 10^8$ GeV and reheating temperatures, T_R, of that order. Other options to lower T_R, and which have more freedom about M_1, include non-thermal production mechanisms [55, 56] where the condition $M_1 \lesssim T_R$ is not required, gravitationally suppressed decay of the inflaton in models of high scale inflation [57] and low scale inflationary models [58, 56, 59].

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5.2 Estimation of Y_B

In order to evaluate the baryon asymmetry, Y_B, we need Y_L, which is given by

$$Y_L = d_L \frac{\varepsilon_1}{g^*}, \qquad d_L = (1-\alpha) d_{B-L}, \tag{5.5}$$

where $g*$ is the effective number of degrees of freedom, for the SM $g^* = 106.75$ and for the MSSM, $g^* = 228.75$. The parameter d is the *dilution factor*, which takes into account the washout effects produced by inverse decays and lepton number violating scatterings. For different models we should integrate numerically the set of Boltzmann equations for the lepton asymmetry Y_L and the asymmetries produced by the right handed Majorana neutrinos, Y_{N_j}. In our case we would like to give just an approximation of Y_L, in order to see whether a possible hierarchy can be realized within the thermal leptogenesis scenario, thus we use the approximation obtained in [60]. There, it has been obtained an *empirical* formula for the value of $Log_{10}(d_{B-L})$, which is taken to be the smallest of the following quantities

$$\begin{aligned}
Log_{10}(d_{B-L}) &= 0.8 Log_{10}(\tilde{m}_1) + 1.7 + 0.05 Log_{10}(M_1^{10}) \\
Log_{10}(d_{B-L}) &= -1.2 - 0.05 Log_{10}(M_1^{10}) \\
Log_{10}(d_{B-L}) &= -(3.8 + Log_{10}(M_1^{10})(Log_{10}(\tilde{m}_1) + 2) - \\
&\quad - \left(5.4 - \frac{2}{3} Log_{10}(M_1^{10})\right)^2 - \frac{3}{2},
\end{aligned} \tag{5.6}$$

where

$$\tilde{m}_1 = \frac{\left(m_D^{\nu\dagger} m_D^{\nu}\right)_{11}}{M_1}, \qquad M_1^{10} = \frac{M_1}{10^{10}\,\mathrm{GeV}}. \tag{5.7}$$

5.3 Relative sign between ε_j and J_{CP} and relation of phases

Now we can study the correlation between the sign of ε_j and the sign of J_{CP}, for the different models presented in section 4. These possible correlations would be satisfied at the scale of the decay of the lightest R-H neutrino. After this the quantities appearing in the CP leptogenesis asymmetries, eq. (5.3), would evolve differently [48].

I. For the models presented in section 4.3, we have that $\mathbf{M_1 < M_2 \ll M_3}$ so we can use eq. (5.4) to evaluate ε_1, but we need to translate it to the symmetry basis, however, in the quantity $H^D = m_{fD}^{\nu\dagger} m_{fD}^{\nu}$ [5] the contribution from the rotation to the base in which the charged leptons are diagonal cancels, thus we have $H_f^D = H_s^D$, and hence:

$$\begin{aligned}
\varepsilon_1 &\approx \frac{3}{16\pi} \frac{|(H_s^D)_{ij}|^2}{v^2 |H_{11}^D|} \frac{M_1}{M_2} \sin(\delta_{\rm L}) \\
\delta_{\rm L} &= 2(\gamma_{12}^D) \\
-\gamma_{12}^D &\equiv \mathrm{Arg}[m_{D21}^{\nu *} m_{D22}^{\nu} + m_{D31}^{\nu *} m_{D32}^{\nu}],
\end{aligned} \tag{5.8}$$

[5] Here the index f corresponds to quantities in the flavour basis and s to quantities in the symmetry basis and $H_b^D = m_{Db}^{\nu\dagger} m_{Db}^{\nu}$ for $b = f, s$.

from now on we drop the index s. As we can see from eqs. (4.27), (4.30), the phase γ_{12}^D is no other than $-\eta_{12}$, then

$$\delta_{\rm L} = -2\eta_{12}\,. \tag{5.9}$$

The phase η_{12} enters into the expression for the CP violation phase associated to neutrino oscillations, $\delta_{\rm O}$, for the different cases presented in example **I** of section 4. Thus in this cases $\delta_{\rm O}$ is related to the phase relevant for leptogenesis, $\delta_{\rm L}$. The exact relation depends on the contribution of the elements diagonalizing the charge lepton mixing matrix, but we will determine whether or not there is a general feature about their relative signs.

(*a*) $\mathbf{s^\nu_{13} \gg s^l_{12}}$. For this case we are considering effectively that $s^l_{12} = 0$ and hence $\gamma^{l\prime}_{12} = 0$, thus we have

$$\delta_{\rm L} = -2\eta_{12}\,. \tag{5.10}$$

For the case $|m^\nu_{D22}c^\nu_{23}| = |m^\nu_{D32}s^\nu_{23}|$[6] we have

$$\delta_{\rm L} = \delta_{\rm O} - \frac{\pi}{2} \to {\rm sign}(\varepsilon_1) = -{\rm sign}(J_{\rm CP}) \tag{5.11}$$

(*b*) $\mathbf{s^\nu_{13} \ll s^l_{12}}$. In this case we compare δ_L to eq. (4.28), for $|m^\nu_{D22}c^\nu_{23}| = |m^\nu_{D32}s^\nu_{23}|$ then we have

$$\delta_{\rm L} \approx 2(\delta_{\rm O} - \gamma^l_{12}) + \pi - 2(\phi^D_{12} - \phi^D_{21}) + 2\theta^l_{23}\sin\chi t^\nu_{23}\,. \tag{5.12}$$

In this case we cannot determine the sign unless for symmetric (anti-symmetric) m^ν_D matrices, in which case we have that $\phi_{12} - \phi_{21} = 0(\pi)$, then

$${\rm sign}(\varepsilon_1) = -{\rm sign}(J_{\rm CP}) \tag{5.13}$$

(*c*) $\mathbf{s^\nu_{13} \approx s^l_{12}\sin^L_{23}}$. In this case we have for $|m^\nu_{D22}c^\nu_{23}| = |m^\nu_{D32}s^\nu_{23}|$

$$\delta_{\rm L} \approx \frac{4}{3}\delta_{\rm O} - \frac{2}{3}\gamma^{l\prime}_{12} + \frac{2\pi}{3} - \frac{2\theta^l_{23}\sin\chi t^\nu_{23}}{3} - \frac{2(\phi^D_{12} - \phi^D_{21})}{3}\,, \tag{5.14}$$

where we cannot determine the sign unless we specify $\gamma^{l\prime}_{12}$. We remark that these cases are not equivalent since they differ in the way the charged lepton mixing angles and the neutrino mixing angles contribute to the $U_{\rm MNS}$ matrix elements, eq. (3.9).

We note that for $\epsilon \approx 0.06$, as was the case presented in [1], then $M_1 \approx 10^8$ GeV, in this case the produced baryon asymmetry, Y_B, is of the order 10^{-14}, which is too small in comparison to the observed values, eq. (5.1). Although this particular realization would not be valid for the thermal leptogenesis scenario considered here,[7] models based on the same structure for masses, eq. (2.6), with $\epsilon \approx 0.2$ and $M_1 \approx 4 \times 10^{10}$ GeV, can produce

[6]Note that in the case of m^ν_{D22} or m^ν_{D32} equal to zero then $\delta_{\rm O} = -2\eta_{12}$ and hence $\delta_{\rm L} = \delta_{\rm O}$, thus ${\rm sign}(\varepsilon_j) = {\rm sign}(J_{CP})$.

[7]We may also think in considering other options for which $M_1 \approx 10^8$ can still be compatible with leptogenesis, such as the one mentioned previously within the context of thermal leptogenesis [3], where there are two quasi-degenerate right handed neutrinos, or we can consider non-thermal leptogenesis scenarios. [55, 61].

a baryon asymmetry of the correct order, eq. (5.1). In [62] the authors have considered a Yukawa neutrino coupling structure similar to eq. (2.6). They have found that is very hard to obtain successful baryogenesis through leptogenesis because in general Y_B is too small. In [63] it was explored the possibility of obtaining the correct amount of baryon asymmetry in the universe in certain types of left-right symmetric theories with a low right-handed scale. It turns out that the Higgs sector of the theory should include one more real scalar field with appropriate self-couplings. In [64, 65] it was explored the possibility of achieving the correct amount of baryon asymmetry in a model with a Yukawa neutrino matrix with a hierarchy for the neutrino Dirac mass matrix with zeroes in the $(1,1)$ and $(1,3)$ positions and similar entries $(2,2)$ and $(2,3)$, such that $Y^{\nu}_{12} \ll Y^{\nu}_{2,2} \ll Y^{\nu}_{3,3} \ll 1$. The conditions on M_R, in order to agree with the correct amount of baryogenesis, were determined there.

II. For the case (i) presented in section 4.3.2 we have $M_1 \ll M_2 \ll M_3$ thus we can use eq. (5.4). For the values $\lambda \approx 0.25$ and $M_1 \approx 10^{13}\,\mathrm{GeV}$ it is possible to produce a baryon asymmetry of the order $Y_B \sim 4 \times 10^{-11}$, which is still compatible with eq. (5.1).

Let us assume first that the matrices of eq. (4.37) are realized in the flavour basis, the relevant phase for leptogenesis is given by $\delta_{\mathrm{L}} = -2\eta_{12}$, eq. (5.9). If we have that $|m^{\nu\prime}_{22}| = |m^{\nu\prime}_{11}|$ then, using eq. (4.18), δ_{L} and the phase appearing in neutrino oscillations, δ_{O}, are simply related by

$$\delta_{\mathrm{L}} = 2\delta_{\mathrm{O}} \to \mathrm{sign}(\varepsilon_1) = \mathrm{sign}(J_{\mathrm{CP}})\,. \tag{5.15}$$

In the symmetry basis we need to take into account the contribution of phases appearing in the diagonalization of charged leptons, eq. (4.50). From eqs. (4.50) we see that the only common phase in δ_{O} and δ_{L} is $\gamma^{l\prime}_{12}$ and we cannot determine the phase unless $\eta_{12} = 0$, and we would have $\mathrm{sign}(\varepsilon_1) = \mathrm{sign}(J_{\mathrm{CP}})$. As we can see for the cases (a), (b), (c), considered here the relation of the signs is not identified unless there are further assumptions and/or the phases coming from charged leptons are identified.

6. Majorana phases and neutrinoless double beta decay

The order of magnitude of the masses of neutrinos and the value of Majorana phases, cannot be determined from neutrino oscillations, the processes from which these parameters can be determined are neutrinoless double beta decay, ($\beta\beta_{\nu 0}$), and tritium beta decay. The Majorana mass term $n_L^T C m^{\nu}_{\mathrm{LL}} n_L$ induces a $\beta\beta_{\nu 0}$ decay ($n\,n \to p\,p\,e^-_L\;e^-_L$) whose amplitude depends on the average neutrino mass

$$\langle m_{\beta\beta} \rangle = \left| \sum_{i=1}^{3} (U^2_{\mathrm{MNS}})_{ei} m_{\nu_i} \right| . \tag{6.1}$$

The Heidelberg-Moscow experiment [66] quotes the range $\langle m_{\beta\beta} \rangle \in (0.11, 0.56)$ eV at 95% confidence level, a result that has been widely criticized and is hoped to be improved, given that neutrino oscillation experiments have sensitivities of order 0.05 eV. The improvement in this measurement it is not expected in the near future, nevertheless, given a possible structure for neutrino mixings it is relevant to obtain its predictions or constraints for the Majorana phases. In the notation of eqs. (3.1), (3.2), (3.3), we have

$$\begin{aligned}\langle m_{\beta\beta}\rangle &= \left|(c_{13}c_{12})^2 e^{2i\sigma_1} m_{\nu_1} + (c_{13}s_{12})^2 m_{\nu_2} + (s_{13})^2 e^{2i(\sigma_2-\delta_{\rm O})} m_{\nu_3}\right| \\ &= \left|(c_{13}c_{12})^2 e^{i(\phi_1-\phi_2)} m_{\nu_1} + (c_{13}s_{12})^2 m_{\nu_2} + (s_{13})^2 e^{i(\phi_3-\phi_2)} m_{\nu_3}\right| \end{aligned} \tag{6.2}$$

so we can analyze the connections for the different cases presented in section 4.

I. In the limit of the strong hierarchy of eq. (4.2) we have $\phi_1 \approx \phi_2$, as we can see from eq. (6.2), nence there is only one relevant phase for $\beta\beta_{\nu 0}$, namely

$$|\phi_{\beta\beta}| = |2(\sigma_2 - \delta_{\rm O})| = |\phi_3 - \phi_2| \,. \tag{6.3}$$

For the case considered in section 4, ϕ_3 and ϕ_2 are given by

$$\phi_2 \approx 2\phi^D_{12}\,, \qquad \phi_3 \approx -2\gamma^{\nu\prime}_{13} + 2\phi^D_{21}\,. \tag{6.4}$$

Inserting eq. (4.6) into eq. (6.4) and substituting in eq. (6.3) we have

$$|\phi_{\beta\beta}| \approx |2\eta_{12}|\,. \tag{6.5}$$

As we can see from eq. (5.9) in the flavour basis, the phase appearing in the amplitude of neutrinoless double beta decay and the phase relevant for leptogenesis are simply related by

$$|\phi_{\beta\beta}| = |\delta_L|\,. \tag{6.6}$$

It is worth mentioning that the result is independent of the approximation $|c^\nu_{23} m^\nu_{D22}| = |s^\nu_{23} m^\nu_{D22}|$, as it provides a simply relation between phases appearing in two very different processes.[8]

II. In this limit $m_{\nu_2} \gtrsim m_{\nu_1} \gg m_{\nu_3}$, thus as we can see from eq. (6.2), the relevant phase for $\beta\beta_{\nu 0}$ is $2\sigma_1 = \phi_2 - \phi_1$. For the case of inverted hierarchies for the mass matrix of eq. (4.1), with neutrino mass matrices satisfying the conditions (4.10), we can see from eqs. (4.13) that the phase $\phi_2 - \phi_1$ is supressed by a small factor $(c^{\nu 2}_{12} - s^{\nu 2}_{12})$, given the similarity of the two masses, $m_{\nu_2} \gtrsim m_{\nu_1}$. Thus the main contribution to this phase is

$$|\phi_{\beta\beta}| = |2\sigma_1| = \left|\frac{(c^{\nu 2}_{12} - s^{\nu 2}_{12})}{f} \sin(2\gamma^{\nu\prime}_{12})\right|, \tag{6.7}$$

where f is a further supression factor.[9] In this case, as it can be seen from eqs. (4.14), (5.9), the relevant phase for neutrinoless double beta decay and the phase for leptogenesis, eq. (5.9), are related by

$$|\phi_{\beta\beta}| = \left|\frac{(c^{\nu 2}_{12} - s^{\nu 2}_{12})}{f} \sin\left(2(\phi^D_{21} - \phi^D_{12}) + \delta_{\rm L} - 2\delta^\nu\right)\right|, \tag{6.8}$$

[8] This result has also been presented in [2].

[9] $f^2 = \sin(2\gamma^{\nu\prime}_{12})^2 \cos(2\theta^\nu_{12})^2 + \left[(3+\cos(4\theta^\nu_{12})) + (m^{\nu\prime 2}_{11} + m^{\nu\prime 2}_{22}) c^{\nu 2}_{12} s^{\nu 2}_{12} / |m^{\nu\prime}_{11} m^{\nu\prime}_{22}|\right]^2$.

where δ^ν is determined by eq. (4.17), in the simplest case $m^{\nu\prime}_{22} = m^{\nu\prime}_{11}$, then $2\delta^\nu$ and $-\delta_{\rm L}$ cancel. If the mass matrix of eq. (4.1), with the conditions (4.10), is realized in the symmetry basis, with small mixings in the leptonic sector, as in example II of section 4.1, then we also need to introduce the contribution of $\gamma^{l\prime}_{12}$ from the mass matrix of charged leptons.

For large values of $\tan\beta$ (≥ 20) order 1 phases ϕ_i experience a small increase ($\sim 2\%$) in their values at electroweak scale, $M_{\rm EW}$, with respect to their values at GUT scale, $M_{\rm G}$. For small values of ϕ_i, the increase on the phases can be as large as $\sim 50\%$. For small values of $\tan\beta$ and for all the values of ϕ_i, the effect of the RGE's on these phases is small ($\lesssim 5\%$). In the flavour basis, there are two contributions to the change in the Dirac CP violating phase δ with respect to $t = \ln(\mu/\mu_O)$, $d\delta/dt$ [48], one is proportional to $m_{\nu_1} m_{\nu_3} \sin(\phi_1 - \delta)$ thus if m_{ν_3} is negligible then this contribution is sub-dominant. In this case, the other contribution, proportional to $m_{\nu_1} m_{\nu_2} \sin(\phi_1 - \phi_2)$, becomes the relevant one, as is the case of the inverted hierarchy presented in section 4.2. Hence, if $\phi_1 - \phi_2$ does not change significatively, the same happens to δ. In the flavour basis, δ is given by eq. (4.17), which is the same combination relevant for neutrinoless double beta decay, eq. (6.5), thus the effects of the RGE's on δ are the same that $\phi_1 - \phi_2$ experiences. In the symmetry basis, when the contribution γ^l_{12} has to be taken into account there is not a significative change in δ because of the hierarchy in Y^l and the weak effects of the RGE's on it. The RGE's produce an increase of $\lesssim 30\%$ in the mass scale of the neutrino masses at $M_{\rm EW}$ with respect to $M_{\rm G}$.

7. Summary and outlook

In order to identify probable symmetries underlying the charged leptons and neutrinos, it is useful to work in the symmetry basis, in which the pattern of the possible (broken) symmetries underlying the leptons are realized, and for which both the neutrinos and leptons mass matrices may not be diagonal. In this way, in general, we can study the contributions that the elements of their diagonalization matrices give to the parameters of the $U_{\rm MNS}$ matrix. We have motivated two successful hierarchies for neutrinos, eq. (4.2) and eq. (4.10), through the family symmetries, $\mathrm{SU}(3)_F$ and $\mathrm{U}(1)_F$ respectively, and studied the contribution from the mixings diagonalizing the charged leptons to the elements of $U_{\rm MNS}$, in particular to the Dirac CP violation phase, $\delta_{\rm O}$, which will be measured in neutrino oscillation experiments.

The contribution to the elements of $U_{\rm MNS}$ coming from the diagonalization of charged leptons may save some of the patterns considered to reproduce the observed mass splittings and mixing angles for neutrinos, eq. (1.1) (which give for example a nearly exact maximal mixing explaining solar neutrinos experiments), in the sense that can receive contributions from the charged leptons. As we have seen, the angle $\theta_{\rm sol}$ may receive contributions from θ^l_{12} which can help $\theta_{\rm sol}$ to deviate from maximality. The same happens with the element $\theta_{\rm rct}$ which can be increased or decreased by taking into account contributions from θ^l_{12}.

A direct relation between the phases appearing in leptogenesis and neutrino oscillation, in general, does not exist. However given that leptogenesis is a very attractive mechanism

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to produce the baryon asymmetry observed in the universe, which is measured to a high precision, eq. (5.1), it is worth-while to look for a connection in models which can describe correctly the observed neutrino mass splittings and mixings. In the leptogenesis scenario the sign of the lepton asymmetry, Y_L, is fixed to reproduce a positive baryon asymmetry, Y_B. If we can write the terms appearing in Y_L, namely ε_j -the asymmetry produced by the decay of the heavy right Majorana neutrinos-, in terms of parameters appearing in neutrino oscillations then it is interesting to determine whether or not there is a relation among the phases appearing in these processes and also if the relative sign of ε_j and J_{CP} may be determined.

We have seen that for hierarchies, eqs. (4.2) and eq. (4.10), describing the mass terms for the low energy neutrinos, m^{ν}_{LL}, there are interesting relations among phases appearing in CP violation for neutrino oscillations and leptogenesis. In cases like these, the phase $\delta_{\rm O}$, when measured in future neutrino oscillation experiments, will help to constraint the possible patterns for neutrino matrices and will determine whether or not these patterns are fully compatible with the leptogenesis, in the sense that they could be able to reproduce both the magnitude and the sign of the baryon asymmetry in the universe, Y_B.

Although the sensitivity of experiments involving neutrinoless double beta decay processes needs to be further increased, we can determine the predictions for the Majorana phases from the models considered here. We have also studied the relations between the Majorana phases and phases appearing in leptogenesis; it is remarkable that in some cases these relations are simple, eqs. (6.6).

Given a successful model describing the masses for leptons, it is interesting to look for predictions relevant to leptogenesis and neutrinoless double beta decay parameters. For example, one can try to identify a possible symmetry to describe the hierarchy $IH1$, eq. (2.5). This is the pseudo-Dirac limit, for which $m_{\nu_1} = -m_{\nu_2}$, and hence the relevant phase for $\beta\beta_{\nu O}$ decay becomes trivial, $2\sigma_1 = 2\pi$.

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A. Diagonalization of hierarchical mass matrices

A.1 Diagonalization of the hermitean matrix $H = mm^{\dagger}$

Writing m in the form $m_{i,j} = |m_{i,j}|e^{i\phi_{ij}}$, the hermitean matrix $H = mm^{\dagger}$ is given by:

$$H = mm^{\dagger} = \begin{pmatrix} H_{11} & H_{12}e^{-i\gamma_{12}} & H_{13}e^{-i\gamma_{13}} \\ H_{12}e^{i\gamma_{12}} & H_{22} & H_{23}e^{-i\gamma_{23}} \\ H_{13}e^{i\gamma_{13}} & H_{23}e^{i\gamma_{23}} & H_{33} \end{pmatrix}, \tag{A.1}$$

where

$$
\begin{aligned}
H_{1j} e^{i\gamma_{1j}} &= m_{11}^* m_{j1} + m_{12}^* m_{j2} + m_{13}^* m_{j3} \\
H_{23} e^{i\gamma_{23}} &= m_{21}^* m_{31} + m_{22}^* m_{32} + m_{23}^* m_{33} \,.
\end{aligned} \tag{A.2}
$$

For the case of quarks and charged leptons ($f = u, d, l$) we use $H^f = m^f m^{f\dagger}$, for the case of the effective mass of neutrinos, in the notation of eq. (2.3), we use

$$
H^\nu = m^{\nu\dagger} m^\nu \,, \tag{A.3}
$$

consequently the elements of H^ν are as in eq. (A.2), with the replacements $H_{ij}^\nu e^{\gamma_{ij}^\nu} \to H_{ij} e^{-\gamma_{ij}}$. H can be diagonalized through rotations and re-phasings which at the end of the procedure can be written in terms of just three rotation angles and six phases, since it is a hermitean matrix, but only three of these are fixed by the elements of H. For hierarchical mass matrices, the first rotation should be in the sector for which a rotation is big (i.e. tan of rotation of order 1), then we can pull out some of the phases and continue making rotations until the off diagonal elements are negligible in comparison with the diagonal ones.

Let us begin with a rotation plus a re-phasing in the 23 sector, making the entries 23 and 32 of H zero:

$$
\begin{aligned}
H' &= V_{23}^\dagger H V_{23} \\
&= \begin{pmatrix} H'_{11} & H'_{12} e^{-i\gamma'_{12}} & H'_{13} e^{-i\gamma'_{13}} \\ H'_{12} e^{i\gamma'_{12}} & H'_{22} & 0 \\ H'_{13} e^{i\gamma'_{13}} & 0 & H'_{33} \end{pmatrix}, \qquad V_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\gamma_{23}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^f & s_{23}^f \\ 0 & -s_{23}^f & c_{23}^f \end{pmatrix}.
\end{aligned} \tag{A.4}
$$

Now we can extract the phases as follows:

$$
H' = \begin{pmatrix} e^{-i\gamma'_{12}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\gamma'_{13}-\gamma'_{12})} \end{pmatrix} \begin{pmatrix} H'_{11} & H'_{12} & H'_{13} \\ H'_{12} & H'_{22} & 0 \\ H'_{13} & 0 & H'_{33} \end{pmatrix} \begin{pmatrix} e^{i\gamma'_{12}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i(\gamma'_{13}-\gamma'_{12})} \end{pmatrix} = P^\dagger H'_r P \,, \tag{A.5}
$$

and we can absorb the phases appearing in H' in the definition of the matrix V_{23} by defining $V'_{23} = V_{23} \mathrm{diag}(e^{-i\gamma'_{12}}, 1, e^{i(\gamma'_{13}-\gamma'_{12})})$. We note that if $H'_{13} \ll H'_{33}$ we may continue rotating the matrix H'_r by a rotation in the 13 sector whose angle $\theta_{13} \approx H'_{13}/H'_{33}$ will be small and thus we have

$$
H''_r = R_{13}^t H'_r R_{13} = \begin{pmatrix} H''_{11} & H''_{12} & 0 \\ H''_{22} & H'_{22} & s_{13} H'_{12} \\ 0 & s_{13} H'_{12} & H''_{33} \end{pmatrix}. \tag{A.6}
$$

If $(s_{13} H'_{12})$ is negligible with respect to the other elements of the matrix then we can continue with a rotation, R_{12} in the 12 sector, which will produce an approximate diagonal matrix if also $(s_{12} H'_{13})$ is negligible with respect to the other elements. Thus the matrix H would be diagonalized only through three successive rotations and re-phasings, which we can identify immediately with the required three mixing angles and phases required to diagonalize any hermitean matrix:

$$
H_{\mathrm{diag}} = R_{12}^t R_{13}^t V_{23}^{\prime\dagger} H V'_{23} R_{13} R_{12} \,, \tag{A.7}
$$

where we define $L^\dagger \equiv R_{12}^t R_{13}^t V_{23}^{\prime\dagger}$.

In this approximation the tangents of the diagonalization angles are given by

$$
\begin{aligned}
t(2\theta^f_{23}) &\approx \frac{2H_{23}}{H_{33}-H_{22}}\,, \\
t(2\theta^f_{13}) &\approx \frac{2H'_{13}}{H'_{33}-H'_{11}} = \frac{2|s^f_{23}H_{12}e^{-i\gamma^f_{12}}+e^{i(\gamma^f_{23}-\gamma^f_{13})}c^f_{23}H_{13}|}{c^{f2}_{23}[H_{33}+t^{f2}_{23}H_{22}+2t_{23}H_{23}]-H_{11}}\,, \\
t(2\theta^f_{12}) &\approx \frac{2H''_{12}}{H''_{22}-H''_{11}} \\
&= \frac{2c^f_{13}|H_{12}c^f_{23}e^{-i\gamma^f_{12}}-e^{i(\gamma^f_{23}-\gamma^f_{13})}s^f_{23}H_{13}|}{c^f_{23}|H_{22}+t^{f2}_{23}H_{33}-2t^f_{23}H_{23}|-[c^{f2}_{13}H_{11}-2c^f_{13}s^f_{13}|c^f_{23}H_{13}e^{i(\gamma^f_{23}+\gamma^f_{12}-\gamma^f_{13})}+H_{12}s^f_{23}|]}\,,
\end{aligned}
\tag{A.8}
$$

where we have also made explicit the relationship between the elements of H'', H' and H, and hence we can identify the phases $\gamma^{f\prime}_{13}$ and $\gamma^{f\prime}_{12}$,

$$
\begin{aligned}
\gamma^{f\prime}_{13} &= \mathrm{Arg}[c^f_{23}H_{13}e^{i(\gamma^f_{13}-\gamma^f_{23})}+s^f_{23}H_{12}e^{i\gamma^f_{12}}]\,, \\
\gamma^{f\prime}_{12} &= \mathrm{Arg}[c^f_{23}H_{12}e^{i\gamma^f_{12}}-s^f_{23}H_{13}e^{i(\gamma^f_{13}-\gamma^f_{23})}]\,.
\end{aligned}
\tag{A.9}
$$

The procedure presented above may be used when diagonalizing canonical or inverted hierarchies of neutrinos (and some hierarchies of charged leptons and quarks), satisfying the following conditions for (strong) *Canonical hierarchies*, $m^f_1 \ll m^f_2 \ll m^f_3$,

$$
m^f_{33} = \mathcal{O}(m^f_{22}, m^f_{23})\,, \quad m^f_{33} \gg \mathcal{O}(m^f_{13})\,, \quad m^f_{13} = \mathcal{O}(m^f_{12})\,, \quad m^f_{11} \ll \mathcal{O}(m^f_{12}, m^f_{23}, m^f_{33})\,,
\tag{A.10}
$$

or for the case of *Inverted hierarchies*, $m^f_1, m^f_2 \gg m^f_3$, such that

$$
m^f_{33} = \mathcal{O}(m^f_{22}, m^f_{23}, m^f_{11}), \qquad m^f_{33} \ll \mathcal{O}m^f_{13}\,, \qquad m^f_{13} = \mathcal{O}(m^f_{12})\,.
\tag{A.11}
$$

We remark that for both cases the hermitean matrix, $H^f = m^f m^{f\dagger}$, has the structure: $H^f_{22} = \mathcal{O}(H^f_{33})$, $H^f_{33} \gg \mathcal{O}(H^f_{13})$, $H^f_{13} = \mathcal{O}(H^f_{12})$. However due to the different hierarchies the tangents of the mixing angles will be dominated by different elements of the original matrices, m, as can be seen in appendix B.

A.2 Diagonalization of H vs. diagonalization of m

If H is diagonalized by L: $H_{\mathrm{diag}} = L^\dagger H L$, eq. (A.7), then L can be written as

$$
L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\gamma_{23}} \end{pmatrix} R_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma^{f\prime}_{12}} & 0 \\ 0 & 0 & e^{i\gamma^{f\prime}_{13}} \end{pmatrix} R_{13}R_{12}e^{-i\gamma^{f\prime}_{12}}\,,
\tag{A.12}
$$

on the other hand if m is diagonalized by L_m, i.e. $m_{\mathrm{diag}} = L^t_m m L_m$, can be expressed in general by

$$
\begin{aligned}
L_m &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & e^{i\beta_1} \end{pmatrix} R_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta_3} & 0 \\ 0 & 0 & 1 \end{pmatrix} R_{13}R_{12} \begin{pmatrix} e^{-i\alpha_0} & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{pmatrix} \\
&= P_1 R_{23} P_2 R_{13} R_{12} P_3\,.
\end{aligned}
\tag{A.13}
$$

Re-arranging the phases we have

$$L_m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\beta_1-\beta_2)} \end{pmatrix} R_{23} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\beta_2+\beta_3)} & 0 \\ 0 & 0 & e^{i\beta_2} \end{pmatrix} R_{13} R_{12} P_3 \,. \tag{A.14}$$

Comparing eq. (A.14) with eq. (A.12) it can be seen that $L_m P_3^\dagger = L$, thus we have the following relations:

$$\gamma_{23} = \beta_1 - \beta_2 \,, \qquad \gamma'_{12} = \beta_2 + \beta_3 \,, \qquad \gamma'_{13} = \beta_2 \,. \tag{A.15}$$

Hence we can express the matrix L_m diagonalizing the mass matrix m in terms of elements of the diagonalization of H and so express the U_{MNS} matrix in these terms. It is useful to notice that L_m can be rewritten as follows

$$\begin{aligned} L_m &= P(1, e^{i\gamma'_{12}}, e^{i(\gamma'_{12}+\gamma_{23})}) R_{23} P(e^{i(\gamma'_{12}-\gamma'_{13})}, 1, 1) R_{13} P(e^{-i(\gamma'_{12}-\gamma'_{13})}, 1, 1) R_{12} \times \\ &\quad \times P(1, 1, e^{i(\gamma'_{12}-\gamma'_{13})}) P_3 \,, \end{aligned} \tag{A.16}$$

because if we were working in the flavour basis then $U_{\mathrm{MNS}} = L_m^\nu$, and hence we can identify the CP violation phase appearing in neutrino oscillations by comparing to the standard parameterization (3.3) so that

$$\delta_{\mathrm{O}} = \gamma_{13}^{\nu\prime} - \gamma_{12}^{\nu\prime} \,. \tag{A.17}$$

We can write the diagonal m_{diag} matrix with phases as:

$$m_{\mathrm{diag}} = L_m^t m L_m = \mathrm{diag}\left(e^{-2i\alpha_1} m_1, e^{-2i\alpha_2} m_2, e^{-2i\alpha_3} m_3\right) , \tag{A.18}$$

where the α_i's make the eigenvalues m_{diag} real, i.e. $\alpha_i = \phi_i/2$ and in the case of neutrinos the two Majorana phases σ_1 and σ_2 can be identified as follows

$$\sigma_1 = 2(\alpha_2 - \alpha_1) = (\phi_1 - \phi_2) \,, \qquad \sigma_2 - \delta_2 = 2(\alpha_2 - \alpha_3) = (\phi_3 - \phi_2) \tag{A.19}$$

in the convention $P(\sigma) = \mathrm{diag}(e^{i\sigma_1}, 1, e^{i\sigma_2})$, as in eq. (3.1).

B. Masses, mixing angles and phases

B.1 Canonical hierarchies

B.1.1 Masses

For both cases, canonical and inverted hierarchies, the diagonal masses $m_i^\nu = |m_i^\nu| e^{i\phi_i}$ are obtained by $L_m^t m L_m$, eq. (A.14), for the case of canonical hierarchies under the conditions (4.2) we have the following approximations, in terms of the elements of the Dirac mass matrix and the heavy Majorana mass matrix,

$$m_{\nu_2} \approx \frac{|m_{D12}^\nu|^2}{M_2 \, |s_{12}^\nu|^2} \,, \qquad m_{\nu_3} \approx \frac{|m_{D21}^\nu|^2 + |m_{D31}^\nu|^2}{M_1} \,. \tag{B.1}$$

In this approximation, the phases of these elements are given by

$$\phi_2 \approx 2\phi_{12}^D \,, \qquad \phi_3 \approx -2\gamma_{13}^{\nu\prime} + 2\phi_{21}^D \,. \tag{B.2}$$

B.1.2 Tangents of the mixing angles

The angle of the rotation R_{23} in terms of the elements of H^{ν} is given by

$$t(2\theta^f_{23}) = \frac{2H_{23}}{H_{33}-H_{22}} = \frac{|m_{13}m^*_{12}+m_{23}m^*_{22}+m_{33}m^*_{32}|}{|m_{33}|^2-|m^2_{22}+|m_{13}|^2-|m_{12}|^2+|m_{23}|^2-|m_{32}|^2} \tag{B.3}$$

where the last term in the denominator vanishes for symmetric matrices.

For the case of canonical hierarchies such that $|m_{12}||m_{11}| \ll |m_{2i}||km_{1i}|$ and $|m_{22}||m_{12}| = \mathcal{O}(|m_{23}||m_{13}|)$, the same expression as eq. (B.3) is obtained for $t(2\theta^f_{23})$ if we begin diagonalizing the matrix m by performing the re-phasing with the diagonal matrix of phases P_2 and the R^f_{23} rotation:

$$m' = R^f_{23}P^t_2 m P_2 R^f_{23}\,. \tag{B.4}$$

This happens because in this case the condition that make zero the entries (23) and (32) is:

$$t(2\theta^f_{23}) = \frac{2|m_{23}|}{|m_{33}|e^{i(\phi_{33}-\phi_{23}+(\beta_1-\beta_2))}-|m_{22}|e^{i(\phi_{22}-\phi_{23}-(\beta_1-\beta 2))}} \tag{B.5}$$

and requiring this quantity to be real, i.e. that

$$|m_{33}|\sin(\phi_{33}-\phi_{23}+(\beta_1-\beta 2)) = |m_{22}|\sin(\phi_{22}-\phi_{23}-(\beta_1-\beta 2))\,, \tag{B.6}$$

is equivalent to

$$\tan(\beta_2-\beta_1) = \frac{|m_{22}|\sin(\phi_{23}-\phi_{22})+|m_{33}|\sin(\phi_{33}-\phi_{23})}{|m_{22}|\cos(\phi_{23}-\phi_{22})+|m_{33}|\cos(\phi_{33}-\phi_{23})} \tag{B.7}$$

which is the same as equation (A.2) for $\gamma_{23} = (\beta_1-\beta_2)$ and the kind of hierarchies of eq. (4.2).

The same kind of equivalences, among the rest of the elements of the diagonalization of H (angles and phases involved in it) and the elements of the diagonalization of m, apply, considering the hierarchies of eq. (4.2). The advantage of diagonalizing $H = mm^{\dagger}$ is that we can extract the relevant phases for CP violation after the first step of diagonalization eq. (A.5).

We present here approximate formulas for the angles diagonalizing the effective neutrino mass matrix in terms of the elements of the neutrino Dirac matrix for the case of hierarchies presented in section 4.1. At leading order

$$\begin{aligned} t^{\nu}_{23} &\approx \frac{|m^{\nu}_{D21}|}{|m^{\nu}_{D31}|}\,, \qquad t^{\nu}_{13} \approx \frac{M_1}{M_2}s^{\nu}_{23}\frac{|m^{\nu *}_{D12}m^{\nu}_{D22}+m^{\nu}_{D13}m^{\nu *}_{D23}|}{|m^{\nu}_{D12}|^2+|m^{\nu}_{D13}|^2}\,,\\ t^{\nu}_{12} &\approx \frac{|m^{\nu}_{D12}|}{c^{\nu}_{23}|m^{\nu}_{D22}|\cos(\phi_{22}-\phi_{12}-\gamma^{\nu\prime}_{12})-s^{\nu}_{23}|m^{\nu}_{D23}|\cos(\phi_{32}-\phi_{12}-\gamma^{\nu}_{23}-\gamma^{\nu\prime}_{12})}\,. \end{aligned} \tag{B.8}$$

B.2 Inverted hierarchies

The inverted Hierarchies for the effective neutrino mass matrix m^ν_{LL}, of the form of eq. (2.5), produce the following hermitean matrices ($H^\nu = m^{\nu\dagger}_{LL} m^\nu_{LL}$), respectively:

$$\mathrm{I}H1: \quad H = \begin{pmatrix} 1+\varepsilon^2 & \frac{3\varepsilon}{\sqrt{2}} & \frac{3\varepsilon}{\sqrt{2}} \\ \frac{3\varepsilon}{\sqrt{2}} & \frac{1}{2}+2\varepsilon^2 & \frac{1}{2}+2\varepsilon^2 \\ \frac{3\varepsilon}{\sqrt{2}} & \frac{1}{2}+2\varepsilon^2 & \frac{1}{2}+2\varepsilon^2 \end{pmatrix}, \quad \mathrm{I}H1: \quad H = \begin{pmatrix} 1+2\varepsilon^2 & 2\varepsilon & 2\varepsilon \\ 2\varepsilon & \frac{1}{2}+\varepsilon^2 & \frac{1}{2}+\varepsilon^2 \\ 2\varepsilon & \frac{1}{2}+\varepsilon^2 & \frac{1}{2}+\varepsilon^2 \end{pmatrix}. \tag{B.9}$$

These matrices satisfy the conditions for the diagonalization process as outlined in section A.1. We can diagonalize them with a diagonalization matrix L_m of the form eq. (A.16), except that now

$$\beta_1 = \gamma^\nu_{23} + \gamma^{\nu\prime}_{13} + \pi\,, \qquad \beta_2 = \gamma^{\nu\prime}_{13}\,, \qquad \beta_3 = \gamma^{\nu\prime}_{12} - \gamma^{\nu\prime}_{13}\,. \tag{B.10}$$

Let us take the case the case of the hierarchy $HI2$, assigning phases of the form ϕ^ν_{ij} and writing $m^\nu = m^\nu_{LL}$. In this case $\mathcal{O}(m^\nu_{33}) = m^\nu_{22}, m^\nu_{23}$, $\mathcal{O}(m^\nu_{33}) \ll m^\nu_{13}$ and $\mathcal{O}(m^\nu_{13}) \ll m^\nu_{12}$. The dominant terms in determining $\tan\theta^\nu_{23}$ are $|m^\nu_{13} m^{\nu *}_{12}|$, so that

$$\tan(2\theta^\nu_{23}) = \frac{|m^\nu_{13} m^{\nu *}_{12}|}{|m^\nu_{12}|^2 - |m^\nu_{13}|^2}\,, \tag{B.11}$$

whose solution for $\tan\theta^\nu_{23}$ is

$$\tan\theta^\nu_{23} = \frac{|m^\nu_{13}|}{|m^\nu_{12}|}\,. \tag{B.12}$$

In this case we can also obtain the phases γ^ν_{ij} from equation (A.2) and the hierarchical conditions of $HI2$, eq. (2.5). The easiest phase to obtain is γ^ν_{23} because it can be determined in the first step of the diagonalization:

$$\gamma^\nu_{23} \approx \phi^\nu_{13} - \phi^\nu_{12}\,. \tag{B.13}$$

After this we can continue with the diagonalization in the sectors 13 and 12. In section 4.2 we have presented the results of this diagonalization in terms of the elements of the Dirac neutrino mass matrix and the right-handed neutrino mass matrix.

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